Instructor's Solutions Manual
Part I

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to accompany

Thomas' Calculus, Early Transcendentals

Tenth Edition

Based on the original work by

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PRELIMINARY CHAPTER

P.1 LINES

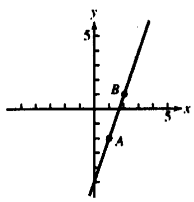
1. (a) $\Delta x = -1 - 1 = -2$
   $\Delta y = -1 - 2 = -3$

   (b) $\Delta x = -1 - (-3) = 2$
   $\Delta y = -2 - 2 = -4$

2. (a) $\Delta x = -8 - (-3) = -5$
   $\Delta y = 1 - 1 = 0$

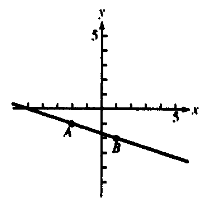
   (b) $\Delta x = 0 - 0 = 0$
   $\Delta y = -2 - 4 = -6$

3. (a)



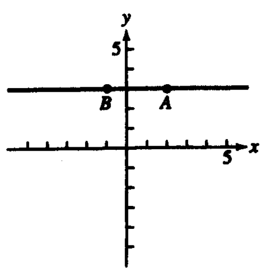
$m = \frac{1-(-2)}{2-1} = \frac{3}{1} = 3$

(b)



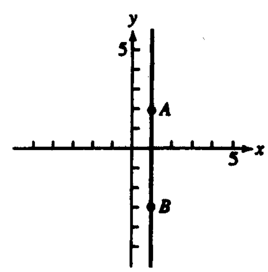
$m = \frac{-2-(-1)}{1-(-2)} = \frac{-1}{3} = -\frac{1}{3}$

4. (a)



$m = \frac{3-3}{-1-2} = \frac{0}{-3} = 0$

(b)



$m = \frac{-3-2}{1-1} = \frac{-5}{0}$ (undefined)

5. (a) $x = 2,\ y = 3$

   (b) $x = -1,\ y = \frac{4}{3}$

6. (a) $x = 0,\ y = -\sqrt{2}$

   (b) $x = -\pi,\ y = 0$

7. (a) $y = 1(x-1)+1$

   (b) $y = -1[x-(-1)]+1 = -1(x+1)+1$

8. (a) $y = 2(x-0)+3$

   (b) $y = -2[x-(-4)]+0 = -2(x+4)+0$

9. (a) $m = \frac{3-0}{2-0} = \frac{3}{2}$

   $y = \frac{3}{2}(x-0)+0$

   $y = \frac{3}{2}x$

   (b) $m = \frac{1-1}{2-1} = \frac{0}{1} = 0$

   $y = 0(x-1)+1$

   $y = 1$

$2y = 3x$

$3x - 2y = 0$

10. (a) $m = \frac{-2-0}{-2-(-2)} = \frac{-2}{0}$ (undefined)

Vertical line: $x = -2$

(b) $m = \frac{-2-1}{2-(-2)} = \frac{-3}{4} = -\frac{3}{4}$

$y = -\frac{3}{4}[x-(-2)]+1$

$4y = -3(x+2)+4$

$4y = -3x-2$

$3x+4y = -2$

11. (a) $y = 3x-2$

(b) $y = -1x+2$ or $y = -x+2$

12. (a) $y = -\frac{1}{2}x-3$

(b) $y = \frac{1}{3}x-1$

13. The line contains $(0,0)$ and $(10,25)$.

$m = \frac{25-0}{10-0} = \frac{25}{10} = \frac{5}{2}$

$y = \frac{5}{2}x$

14. The line contains $(0,0)$ and $(5,2)$.

$m = \frac{2-0}{5-0} = \frac{2}{5}$

$y = \frac{2}{5}x$

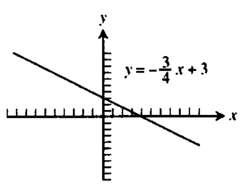
15. (a) $3x+4y = 12$

$4y = -3x+12$

$y = -\frac{3}{4}x+3$

i) Slope: $-\frac{3}{4}$

ii) y-intercept: 3

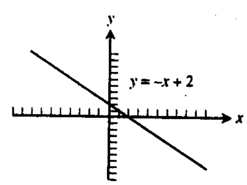


(b) $x+y = 2$

$y = -x+2$

i) Slope: -1

ii) y-intercept: 2



16. (a) $\frac{x}{3}+\frac{y}{4} = 1$

$\frac{y}{4} = -\frac{x}{3}+1$

$y = -\frac{4}{3}x+4$

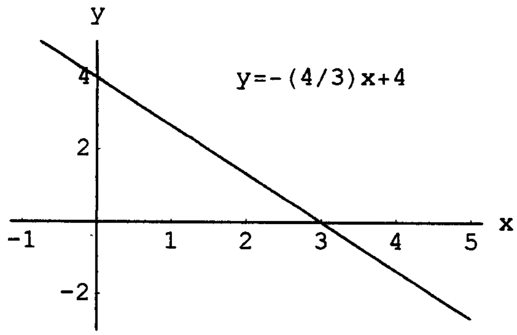
i) Slope: $-\frac{4}{3}$

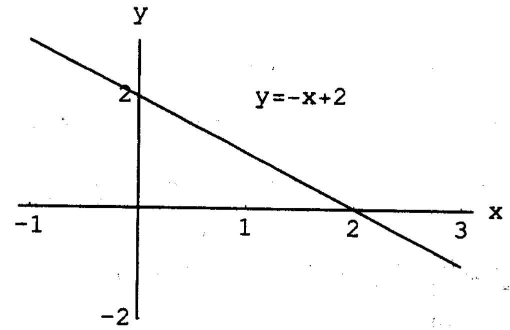
ii) y-intercept: 4

(b) $y = 2x+4$

i) Slope: 2

ii) y-intercept: 4





17. (a) i) The desired line has slope -1 and passes through $(0,0)$: $y = -1(x-0)+0$ or $y = -x$.
ii) The desired line has slope $\frac{-1}{-1} = 1$ and passes through $(0,0)$: $y = 1(x-0)+0$ or $y = x$.
(b) i) The given equation is equivalent to $y = -2x+4$. The desired line has slope -2 and passes through $(-2,2)$: $y = -2(x+2)+2$ or $y = -2x-2$.
ii) The desired line has slope $\frac{-1}{-2} = \frac{1}{2}$ and passes through $(-2,2)$: $y = \frac{1}{2}(x+2)+2$ or $y = \frac{1}{2}x+3$.

18. (a) i) The given line is vertical, so we seek a vertical line through $(-2,4)$: $x = -2$.
ii) We seek a horizontal line through $(-2,4)$: $y = 4$.
(b) i) The given line is horizontal, so we seek a horizontal line through $\left(-1,\frac{1}{2}\right)$: $y = \frac{1}{2}$.
ii) We seek a vertical line through $\left(-1,\frac{1}{2}\right)$: $x = -1$.

19. $m = \frac{9-2}{3-1} = \frac{7}{2}$

$f(x) = \frac{7}{2}(x-1)+2 = \frac{7}{2}x - \frac{3}{2}$

Check: $f(5) = \frac{7}{2}(5) - \frac{3}{2} = 16$, as expected.

Since $f(x) = \frac{7}{2}x - \frac{3}{2}$, we have $m = \frac{7}{2}$ and $b = -\frac{3}{2}$.

20. $m = \frac{-4-(-1)}{4-2} = \frac{-3}{2} = -\frac{3}{2}$

$f(x) = -\frac{3}{2}(x-2)+(-1) = -\frac{3}{2}x+2$

Check: $f(6) = -\frac{3}{2}(6)+2 = -7$, as expected.

Since $f(x) = -\frac{3}{2}x+2$, we have $m = -\frac{3}{2}$ and $b = 2$.

21. $-\frac{2}{3} = \frac{y-3}{4-(-2)}$

$-\frac{2}{3}(6) = y-3$

$-4 = y-3$

$-1 = y$

22. $2 = \frac{2-(-2)}{x-(-8)}$

$2(x+8) = 4$

$x+8 = 2$

$x = -6$

23. $y = 1\cdot(x-3)+4$
$y = x-3+4$
$y = x+1$
This is the same as the equation obtained in Example 5.

24. (a) When $y = 0$, we have $\frac{x}{c} = 1$, so $x = c$.

When $x = 0$, we have $\frac{y}{d} = 1$, so $y = d$.

(b) When $y = 0$, we have $\frac{x}{c} = 2$, so $x = 2c$.

When $x = 0$, we have $\frac{y}{d} = 2$, so $y = 2d$.
The x-intercept is 2c and the y-intercept is 2d.

25. (a) The given equations are equivalent to $y = -\frac{2}{k}x + \frac{3}{k}$ and $y = -x + 1$, respectively, so the slopes are $-\frac{2}{k}$ and -1. The lines are parallel when $-\frac{2}{k} = -1$, so $k = 2$.

(b) The lines are perpendicular when $-\frac{2}{k} = \frac{-1}{-1}$, so $k = -2$.

26. (a) $m \approx \frac{68 - 69.5}{0.4 - 0} = \frac{-1.5}{0.4} = -3.75$ degrees/inch

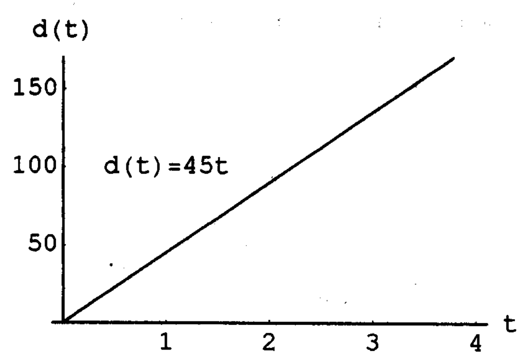
(b) $m \approx \frac{10 - 68}{4 - 0.4} = \frac{-58}{3.6} \approx -16.1$ degrees/inch

(c) $m \approx \frac{5 - 10}{4.7 - 4} = \frac{-5}{0.7} = -7.1$ degrees/inch

(d) Best insulator: Fiberglass insulation
Poorest insulator: Gypsum wallboard
The best insulator will have the largest temperature change per inch, because that will allow larger temperature differences on opposite sides of thinner layers.

27. Slope: $k = \frac{\Delta p}{\Delta d} = \frac{10.94 - 1}{100 - 0} = \frac{9.94}{100} = 0.0994$ atmospheres per meter
At 50 meters, the pressure is $p = 0.0994(50) + 1 = 5.97$ atmospheres.

28. (a) $d(t) = 45t$
(b)



(c) The slope is 45, which is the speed in miles per hour.
(d) Suppose the car has been traveling 45 mph for several hours when it is first observed at point P at time $t = 0$.
(e) The car starts at time $t = 0$ at a point 30 miles past P.

29. (a) Suppose x°F is the same as x°C.

$x = \frac{9}{5}x + 32$

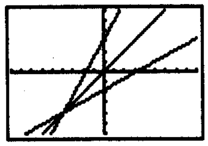
$\left(1 - \frac{9}{5}\right)x = 32$

$-\frac{4}{5}x = 32$

$x = -40$

Yes, −40°F is the same as −40°C.

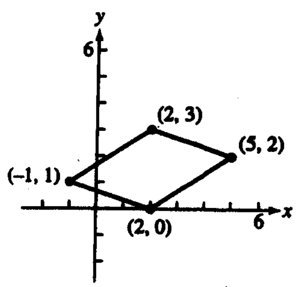
(b)

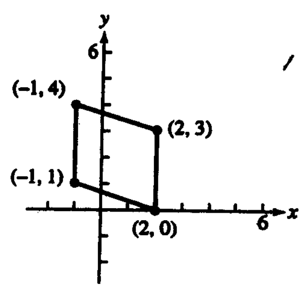


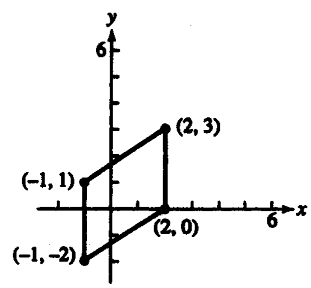
[−90, 90] by [−60, 60]

It is related because all three lines pass through the point $(-40, -40)$ where the Fahrenheit and Celsius temperatures are the same.

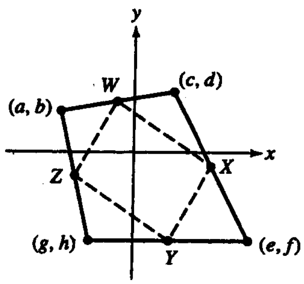
30. The coordinates of the three missing vertices are $(5, 2)$, $(-1, 4)$ and $(-1, -2)$, as shown below.







31.



Suppose that the vertices of the given quadrilateral are (a, b), (c, d), (e, f), and (g, h). Then the midpoints of the consecutive sides are $W\left(\frac{a+c}{2}, \frac{b+d}{2}\right)$, $X\left(\frac{c+e}{2}, \frac{d+f}{2}\right)$, $Y\left(\frac{e+g}{2}, \frac{f+h}{2}\right)$, and $Z\left(\frac{g+a}{2}, \frac{h+b}{2}\right)$. When these four points are connected, the slopes of the sides of the resulting figure are:

WX: $\dfrac{\frac{d+f}{2}-\frac{b+d}{2}}{\frac{c+e}{2}-\frac{a+c}{2}}=\frac{f-b}{e-a}$

XY: $\dfrac{\frac{f+h}{2}-\frac{d+f}{2}}{\frac{e+g}{2}-\frac{c+e}{2}}=\frac{h-d}{g-c}$

ZY: $\dfrac{\frac{f+h}{2}-\frac{h+b}{2}}{\frac{e+g}{2}-\frac{g+a}{2}}=\frac{f-b}{e-a}$

WZ: $\dfrac{\frac{h+b}{2}-\frac{b+d}{2}}{\frac{g+a}{2}-\frac{a+c}{2}}=\frac{h-d}{g-c}$

Opposite sides have the same slope and are parallel.

32. The radius through (3, 4) has slope $\frac{4-0}{3-0}=\frac{4}{3}$.

The tangent line is perpendicular to this radius, so its slope is $\frac{-1}{4/3}=-\frac{3}{4}$. We seek the line of slope $-\frac{3}{4}$ that passes through (3, 4).

$y=-\frac{3}{4}(x-3)+4$

$y=-\frac{3}{4}x+\frac{9}{4}+4$

$y=-\frac{3}{4}x+\frac{25}{4}$

33. (a) The equation for line L can be written as

$y=-\frac{A}{B}x+\frac{C}{B}$, so its slope is $-\frac{A}{B}$. The perpendicular line has slope $\frac{-1}{-A/B}=\frac{B}{A}$ and passes through (a, b), so its equation is $y=\frac{B}{A}(x-a)+b$.

(b) Substituting $\frac{B}{A}(x-a)+b$ for y in the equation for line L gives:

$Ax+B\left[\frac{B}{A}(x-a)+b\right]=C$

$$A^2x + B^2(x-a) + ABb = AC$$
$$(A^2+B^2)x = B^2a + AC - ABb$$
$$x = \frac{B^2a + AC - ABb}{A^2+B^2}$$

Substituting the expression for x in the equation for line L gives:

$$A\left(\frac{B^2a + AC - ABb}{A^2+B^2}\right) + By = C$$

$$By = \frac{-A(B^2a + AC - ABb)}{A^2+B^2} + \frac{C(A^2+B^2)}{A^2+B^2}$$

$$By = \frac{-AB^2a - A^2C + A^2Bb + A^2C + B^2C}{A^2+B^2}$$

$$By = \frac{A^2Bb + B^2C - AB^2a}{A^2+B^2}$$

$$y = \frac{A^2b + BC - ABa}{A^2+B^2}$$

The coordinates of Q are $\left(\frac{B^2a + AC - ABb}{A^2+B^2}, \frac{A^2b + BC - ABa}{A^2+B^2}\right)$.

(c) Distance $= \sqrt{(x-a)^2 + (y-b)^2}$

$$= \sqrt{\left(\frac{B^2a + AC - ABb}{A^2+B^2} - a\right)^2 + \left(\frac{A^2b + BC - ABa}{A^2+B^2} - b\right)^2}$$

$$= \sqrt{\left(\frac{B^2a + AC - ABb - a(A^2+B^2)}{A^2+B^2}\right)^2 + \left(\frac{A^2b + BC - ABa - b(A^2+B^2)}{A^2+B^2}\right)^2}$$

$$= \sqrt{\left(\frac{AC - ABb - A^2a}{A^2+B^2}\right)^2 + \left(\frac{BC - ABa - B^2b}{A^2+B^2}\right)^2}$$

$$= \sqrt{\left(\frac{A(C - Bb - Aa)}{A^2+B^2}\right)^2 + \left(\frac{B(C - Aa - Bb)}{A^2+B^2}\right)^2}$$

$$= \sqrt{\frac{A^2(C - Aa - Bb)^2}{(A^2+B^2)^2} + \frac{B^2(C - Aa - Bb)^2}{(A^2+B^2)^2}}$$

$$= \sqrt{\frac{(A^2+B^2)(C - Aa - Bb)^2}{(A^2+B^2)^2}}$$

$$= \sqrt{\frac{(C - Aa - Bb)^2}{A^2+B^2}}$$

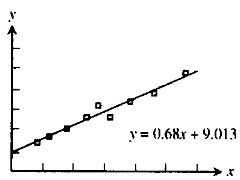
$$= \frac{|C - Aa - Bb|}{\sqrt{A^2 + B^2}}$$
$$= \frac{|Aa + Bb - C|}{\sqrt{A^2 + B^2}}$$

34. The line of incidence passes through $(0,1)$ and $(1,0)$ $\Rightarrow$ The line of reflection passes through $(1,0)$ and $(2,1)$ $\Rightarrow m = \frac{1-0}{2-1} = 1 \Rightarrow y - 0 = 1(x-1) \Rightarrow y = x - 1$ is the line of reflection.

35. $m = \frac{37.1}{100} = \frac{14}{\Delta x} \Rightarrow \Delta x = \frac{14}{.371}$. Therefore, distance between first and last rows is $\sqrt{(14)^2 + \left(\frac{14}{.371}\right)^2} \approx 40.25$ ft.

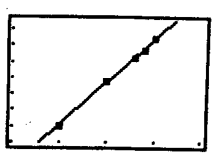
36. (a) $(-1,4)$ (b) $(3,-2)$ (c) $(5,2)$ (d) $(0,x)$
(e) $(-y,0)$ (f) $(-y,x)$ (g) $(3,-10)$

37. (a) $y = 0.680x + 9.013$
(b) The slope is 0.68. It represents the approximate average weight gain in pounds per month.
(c)



(d) When $x = 30$, $y \approx 0.680(30) + 9.013 = 29.413$.
She weighs about 29 pounds.

38. (a) $y = 1060.4233x - 2{,}077{,}548.669$
(b) The slope is 1060.4233. It represents the approximate rate of increase in earnings in dollars per year.
(c)



[1975, 1995] by [20,000, 35,000]

(d) When $x = 2000$, $y \approx 1060.4233(2000) - 2{,}077{,}548.669 \approx 43{,}298$.
In 2000, the construction workers' average annual compensation will be about \$43,298.

39. (a) $y = 5632x - 11{,}080{,}280$
(b) The rate at which the median price is increasing in dollars per year
(c) $y = 2732x - 5{,}362{,}360$
(d) The median price is increasing at a rate of about \$5632 per year in the Northeast, and about \$2732 per year in the Midwest. It is increasing more rapidly in the Northeast.

P.2 FUNCTIONS AND GRAPHS

1. base = x; $(\text{height})^2 + \left(\frac{x}{2}\right)^2 = x^2 \Rightarrow \text{height} = \frac{\sqrt{3}}{2}x$; area is $a(x) = \frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}(x)\left(\frac{\sqrt{3}x}{2}\right) = \frac{\sqrt{3}}{4}x^2$; perimeter is $p(x) = x + x + x = 3x$.

2. s = side length $\Rightarrow s^2 + s^2 = d^2 \Rightarrow s = \frac{d}{\sqrt{2}}$; and area is $a = s^2 \Rightarrow a = \frac{1}{2}d^2$

3. Let D = diagonal of a face of the cube and ℓ = the length of an edge. Then $\ell^2 + D^2 = d^2$ and (by Exercise 2) $D^2 = 2\ell^2 \Rightarrow 3\ell^2 = d^2 \Rightarrow \ell = \frac{d}{\sqrt{3}}$. The surface area is $6\ell^2 = \frac{6d^2}{3} = 2d^2$ and the volume is $\ell^3 = \left(\frac{d}{\sqrt{3}}\right)^{3/2}$ $= \frac{d^3}{3\sqrt{3}}$.

4. The coordinates of P are $(x, \sqrt{x})$ so the slope of the line joining P to the origin is $m = \frac{\sqrt{x}}{x} = \frac{1}{\sqrt{x}}$ $(x > 0)$. Thus $\sqrt{x} = \frac{1}{m}$ and the x-coordinate of P is $x = \frac{1}{m^2}$; the y-coordinate of P is $y = \frac{1}{m}$.

5. (a) Not the graph of a function of x since it fails the vertical line test.
   (b) Is the graph of a function of x since any vertical line intersects the graph at most once.

6. (a) Not the graph of a function of x since it fails the vertical line test.
   (b) Not the graph of a function of x since it fails the vertical line test.

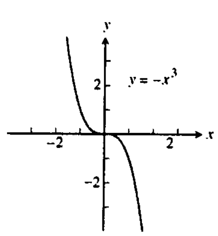
7. (a) domain $= (-\infty, \infty)$; range $= [1, \infty)$
   (b) domain $= [0, \infty)$; range $= (-\infty, 1]$

8. (a) domain $= (0, \infty)$; y in range $\Rightarrow y = \frac{1}{\sqrt{t}}$, $t > 0 \Rightarrow y^2 = \frac{1}{t}$ and $y > 0 \Rightarrow$ y can be any positive real number $\Rightarrow$ range $= (0, \infty)$.
   (b) domain $= [0, \infty)$; y in range $\Rightarrow y = \frac{1}{1+\sqrt{t}}$, $t > 0$. If $t = 0$, then $y = 1$ and as t increases, y becomes a smaller and smaller positive real number $\Rightarrow$ range $= (0, 1]$.

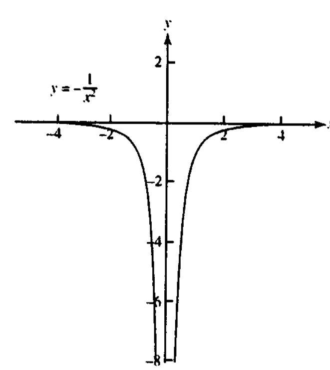
9. $4 - z^2 = (2 - z)(2 + z) \geq 0 \Leftrightarrow z \in [-2, 2]$ = domain. Largest value is $g(0) = \sqrt{4} = 2$ and smallest value is $g(-2) = g(2) = \sqrt{0} = 0 \Rightarrow$ range $= [0, 2]$.

10. domain $= (-\infty, \infty)$; range $= (-\infty, \infty)$

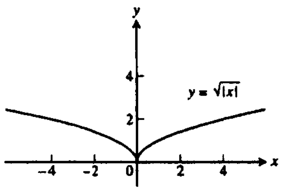
11. (a) Symmetric about the origin



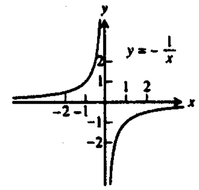
   (b) Symmetric about the y-axis



12. (a) Symmetric about the y-axis

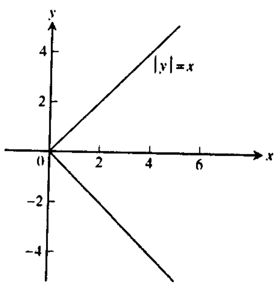


(b) Symmetric about the origin

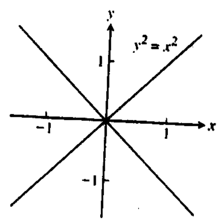


13. Neither graph passes the vertical line test

(a)

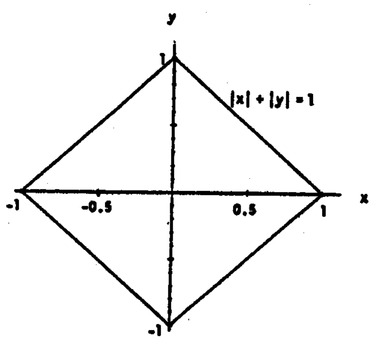


(b)

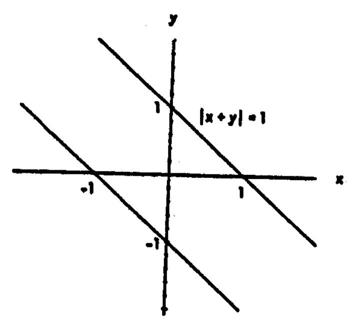


14. Neither graph passes the vertical line test

(a)



(b)



$$|x+y| = 1 \Leftrightarrow \left\{\begin{matrix} x+y=1 \\ \text{or} \\ x+y=-1 \end{matrix}\right\} \Leftrightarrow \left\{\begin{matrix} y=1-x \\ \text{or} \\ y=-1-x \end{matrix}\right\}$$

15. (a) even
    (b) odd

16. (a) even
    (b) neither

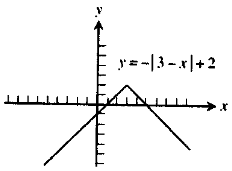
17. (a) odd
    (b) even

18. (a) even
    (b) odd

19. (a) neither
    (b) even

20. (a) even
    (b) even

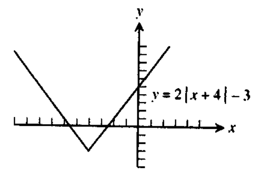
21. (a)



Note that $f(x) = -|x - 3| + 2$, so its graph is the graph of the absolute value function reflected across the x-axis and then shifted 3 units right and 2 units upward.

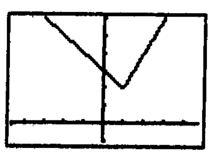
$(-\infty, \infty)$
$(-\infty, 2]$

(b) The graph of $f(x)$ is the graph of the absolute value function stretched vertically by a factor of 2 and then shifted 4 units to the left and 3 units downward



$(-\infty, \infty)$ or all real numbers
$[-3, \infty)$

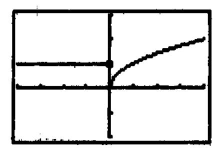
22. (a)



[−4.7, 4.7] by [−1, 6]

$(-\infty, \infty)$ or all real numbers
$[2, \infty)$

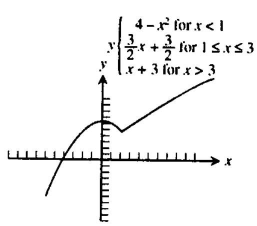
(b)



[−4, 4] by [−2, 3]

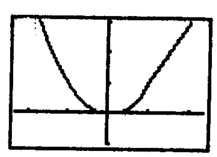
$(-\infty, \infty)$ or all real numbers
$[0, \infty)$

23. (a)



(b) $(-\infty, \infty)$ or all real numbers
(c) $(-\infty, \infty)$ or all real numbers

24. (a)



[−2.35, 2.35] by [−1, 3]

(b) $(-\infty, \infty)$ or all real numbers
(c) $[0, \infty)$

25. Because if the vertical line test holds, then for each x-coordinate, there is at most one y-coordinate giving a point on the curve. This y-coordinate corresponds to the value assigned to the x-coordinate. Since there is only one y-coordinate, the assignment is unique.

26. If the curve is not $y = 0$, there must be a point (x, y) on the curve where $y \neq 0$. That would mean that (x, y) and $(x, -y)$ are two different points on the curve and it is not the graph of a function, since it fails the vertical line test.

27. (a) Line through (0,0) and (1,1): $y = x$
Line through (1,1) and (2,0): $y = -x + 2$

$$f(x) = \begin{cases} x, & 0 \le x \le 1 \\ -x+2, & 1 < x \le 2 \end{cases}$$

(b) $$f(x) = \begin{cases} 2, & 0 \le x < 1 \\ 0, & 1 \le x < 2 \\ 2, & 2 \le x < 3 \\ 0, & 3 \le x \le 4 \end{cases}$$

(c) Line through (0,2) and (2,0): $y = -x + 2$

Line through (2,1) and (5,0): $m = \frac{0-1}{5-2} = \frac{-1}{3} = -\frac{1}{3}$, so $y = -\frac{1}{3}(x-2) + 1 = -\frac{1}{3}x + \frac{5}{3}$

$$f(x) = \begin{cases} -x+2, & 0 < x \le 2 \\ -\frac{1}{3}x + \frac{5}{3}, & 2 < x \le 5 \end{cases}$$

(d) Line through (−1,0) and (0,−3): $m = \frac{-3-0}{0-(-1)} = -3$, so $y = -3x - 3$

Line through (0,3) and (2,−1): $m = \frac{-1-3}{2-0} = \frac{-4}{2} = -2$, so $y = -2x + 3$

$$f(x) = \begin{cases} -3x-3, & -1 < x \le 0 \\ -2x+3, & 0 < x \le 2 \end{cases}$$

28. (a) Line through (−1,1) and (0,0): $y = -x$

Line through (0,1) and (1,1): $y = 1$

Line through (1,1) and (3,0): $m = \frac{0-1}{3-1} = \frac{-1}{2} = -\frac{1}{2}$, so $y = -\frac{1}{2}(x-1) + 1 = -\frac{1}{2}x + \frac{3}{2}$

$$f(x) = \begin{cases} -x, & -1 \le x < 0 \\ 1, & 0 < x \le 1 \\ -\frac{1}{2}x + \frac{3}{2}, & 1 < x < 3 \end{cases}$$

(b) Line through (−2,−1) and (0,0): $y = \frac{1}{2}x$

Line through (0,2) and (1,0): $y = -2x + 2$

Line through (1,−1) and (3,−1): $y = -1$

$$f(x) = \begin{cases} \frac{1}{2}x, & -2 \le x \le 0 \\ -2x+2, & 0 < x \le 1 \\ -1, & 1 < x \le 3 \end{cases}$$

(c) Line through $\left(\frac{T}{2}, 0\right)$ and $(T, 1)$: $m = \frac{1-0}{T-(T/2)} = \frac{2}{T}$, so $y = \frac{2}{T}\left(x - \frac{T}{2}\right) + 0 = \frac{2}{T}x - 1$

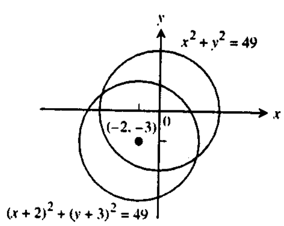
$$f(x) = \begin{cases} 0, & 0 \le x \le \frac{T}{2} \\ \frac{2}{T}x - 1, & \frac{T}{2} < x \le T \end{cases}$$

(d) $$f(x) = \begin{cases} A, & 0 \le x < \frac{T}{2} \\ -A, & \frac{T}{2} \le x < T \\ A, & T \le x < \frac{3T}{2} \\ -A, & \frac{3T}{2} \le x \le 2T \end{cases}$$

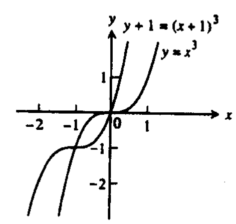
29. (a) Position 4 (b) Position 1 (c) Position 2 (d) Position 3

30. (a) $y = -(x-1)^2 + 4$ (b) $y = -(x+2)^2 + 3$ (c) $y = -(x+4)^2 - 1$ (d) $y = -(x-2)^2$

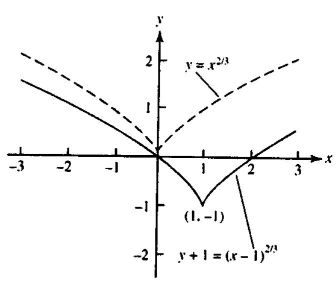
31.



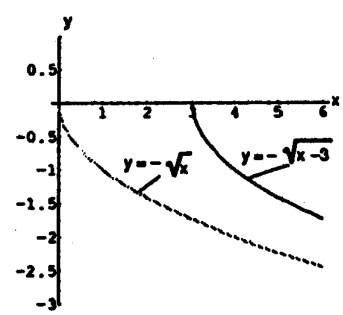
32.



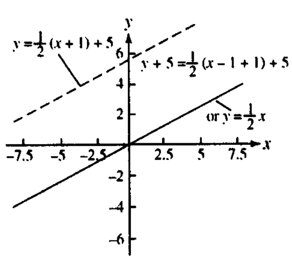
33.



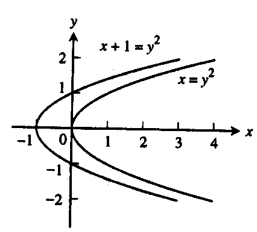
34.



35.



36.



37. (a) $f(g(0)) = f(-3) = 2$
    (b) $g(f(0)) = g(5) = 22$
    (c) $f(g(x)) = f(x^2-3) = x^2-3+5 = x^2+2$
    (d) $g(f(x)) = g(x+5) = (x+5)^2-3 = x^2+10x+22$
    (e) $f(f(-5)) = f(0) = 5$
    (f) $g(g(2)) = g(1) = -2$
    (g) $f(f(x)) = f(x+5) = (x+5)+5 = x+10$
    (h) $g(g(x)) = g(x^2-3) = (x^2-3)^2-3 = x^4-6x^2+6$

38. (a) $f\left(g\left(\frac{1}{2}\right)\right) = f\left(\frac{2}{3}\right) = -\frac{1}{3}$
    (b) $g\left(f\left(\frac{1}{2}\right)\right) = g\left(-\frac{1}{2}\right) = 2$
    (c) $f(g(x)) = f\left(\frac{1}{x+1}\right) = \frac{1}{x+1} - 1 = \frac{-x}{x+1}$
    (d) $g(f(x)) = g(x-1) = \frac{1}{(x-1)+1} = \frac{1}{x}$
    (e) $f(f(2)) = f(1) = 0$
    (f) $g(g(2)) = g\left(\frac{1}{3}\right) = \frac{1}{\frac{4}{3}} = \frac{3}{4}$
    (g) $f(f(x)) = f(x-1) = (x-1)-1 = x-2$
    (h) $g(g(x)) = g\left(\frac{1}{x+1}\right) = \frac{1}{\frac{1}{x+1}+1} = \frac{x+1}{x+2}$ $(x \neq -1 \text{ and } x \neq -2)$

39. (a) $u(v(f(x))) = u\left(v\left(\frac{1}{x}\right)\right) = u\left(\frac{1}{x^2}\right) = 4\left(\frac{1}{x}\right)^2 - 5 = \frac{4}{x^2} - 5$
    (b) $u(f(v(x))) = u\left(f\left(x^2\right)\right) = u\left(\frac{1}{x^2}\right) = 4\left(\frac{1}{x^2}\right) - 5 = \frac{4}{x^2} - 5$
    (c) $v(u(f(x))) = v\left(u\left(\frac{1}{x}\right)\right) = v\left(4\left(\frac{1}{x}\right) - 5\right) = \left(\frac{4}{x} - 5\right)^2$
    (d) $v(f(u(x))) = v(f(4x-5)) = v\left(\frac{1}{4x-5}\right) = \left(\frac{1}{4x-5}\right)^2$
    (e) $f(u(v(x))) = f\left(u\left(x^2\right)\right) = f\left(4\left(x^2\right) - 5\right) = \frac{1}{4x^2-5}$
    (f) $f(v(u(x))) = f(v(4x-5)) = f\left((4x-5)^2\right) = \frac{1}{(4x-5)^2}$

40. (a) $h(g(f(x))) = h\left(g\left(\sqrt{x}\right)\right) = h\left(\frac{\sqrt{x}}{4}\right) = 4\left(\frac{\sqrt{x}}{4}\right) - 8 = \sqrt{x} - 8$
    (b) $h(f(g(x))) = h\left(f\left(\frac{x}{4}\right)\right) = h\left(\sqrt{\frac{x}{4}}\right) = 4\sqrt{\frac{x}{4}} - 8 = 2\sqrt{x} - 8$

(c) $g(h(f(x))) = g(h(\sqrt{x})) = g(4\sqrt{x} - 8) = \frac{4\sqrt{x} - 8}{4} = \sqrt{x} - 2$

(d) $g(f(h(x))) = g(f(4x - 8)) = g(\sqrt{4x - 8}) = \frac{\sqrt{4x - 8}}{4} = \frac{\sqrt{x - 2}}{2}$

(e) $f(g(h(x))) = f(g(4x - 8)) = f\left(\frac{4x - 8}{4}\right) = f(x - 2) = \sqrt{x - 2}$

(f) $f(h(g(x))) = f\left(h\left(\frac{x}{4}\right)\right) = f\left(4\left(\frac{x}{4}\right) - 8\right) = f(x - 8) = \sqrt{x - 8}$

41. (a) $y = g(f(x))$
(b) $y = j(g(x))$
(c) $y = g(g(x))$
(d) $y = j(j(x))$
(e) $y = g(h(f(x)))$
(f) $y = h(j(f(x)))$

42. (a) $y = f(j(x))$
(b) $y = h(g(x)) = g(h(x))$
(c) $y = h(h(x))$
(d) $y = f(f(x))$
(e) $y = j(g(f(x)))$
(f) $y = g(f(h(x)))$

43. (a) Since $(f \circ g)(x) = \sqrt{g(x) - 5} = \sqrt{x^2 - 5}$, $g(x) = x^2$.

(b) Since $(f \circ g)(x) = 1 + \frac{1}{g(x)} = x$, we know that $\frac{1}{g(x)} = x - 1$, so $g(x) = \frac{1}{x - 1}$.

(c) Since $(f \circ g)(x) = f\left(\frac{1}{x}\right) = x$, $f(x) = \frac{1}{x}$.

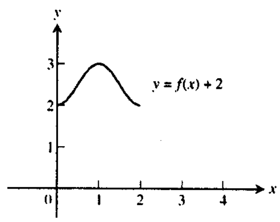
(d) Since $(f \circ g)(x) = f(\sqrt{x}) = |x|$, $f(x) = x^2$.
The completed table is shown. Note that the absolute value sign in part (d) is optional.

g(x)	f(x)	(f ∘ g)(x)
x^2	$\sqrt{x - 5}$	$\sqrt{x^2 - 5}$
$\frac{1}{x - 1}$	$1 + \frac{1}{x}$	$x, x \neq -1$
$\frac{1}{x}$	$\frac{1}{x}$	$x, x \neq 0$
$\sqrt{x}$	x^2	$\|x\|, x \geq 0$

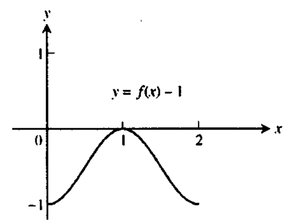
44.

	g(x)	f(x)	(f ∘ g)(x)
(a)	$x - 7$	$\sqrt{x}$	$\sqrt{x - 7}$
(b)	$x + 2$	$3x$	$3(x + 2) = 3x + 6$
(c)	x^2	$\sqrt{x - 5}$	$\sqrt{x^2 - 5}$
(d)	$\frac{x}{x - 1}$	$\frac{x}{x - 1}$	$\frac{\frac{x}{x-1}}{\frac{x}{x-1} - 1} = \frac{x}{x - (x - 1)} = x$
(e)	$\frac{1}{x - 1}$	$1 + \frac{1}{x}$	$1 + \frac{1}{\frac{1}{x-1}} = 1 + (x - 1) = x$
(f)	$\frac{1}{x}$	$\frac{1}{x}$	$\frac{1}{\frac{1}{x}} = x$

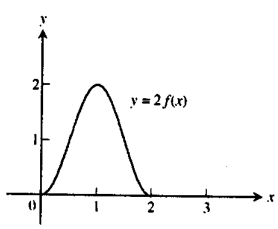
45. (a) domain: $[0, 2]$; range: $[2, 3]$



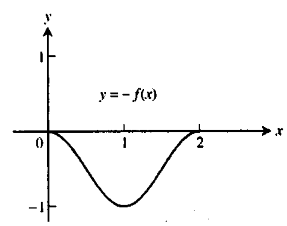
(b) domain: $[0, 2]$; range: $[-1, 0]$



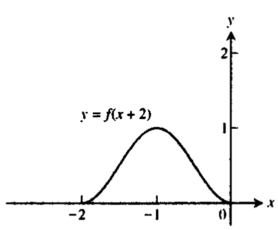
(c) domain: $[0, 2]$; range: $[0, 2]$



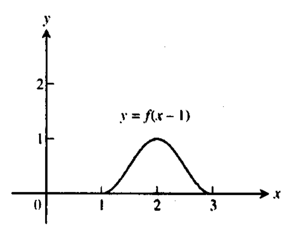
(d) domain: $[0, 2]$; range: $[-1, 0]$



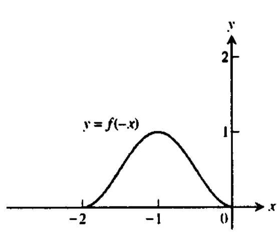
(e) domain: $[-2, 0]$; range: $[0, 1]$



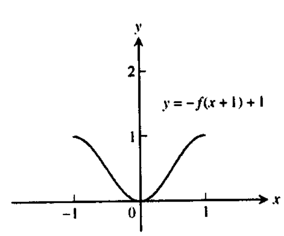
(f) domain: $[1, 3]$; range: $[0, 1]$



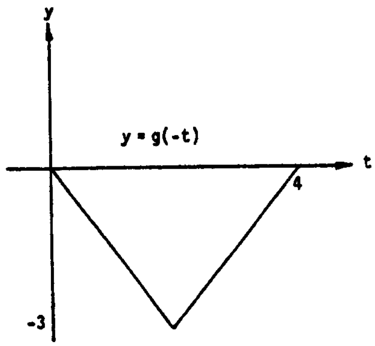
(g) domain: $[-2, 0]$; range: $[0, 1]$



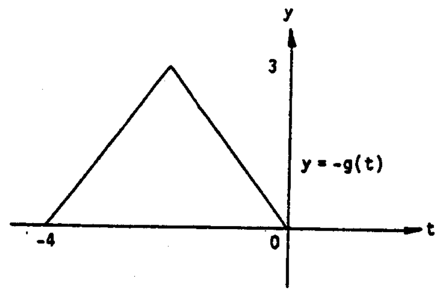
(h) domain: $[-1, 1]$; range: $[0, 1]$



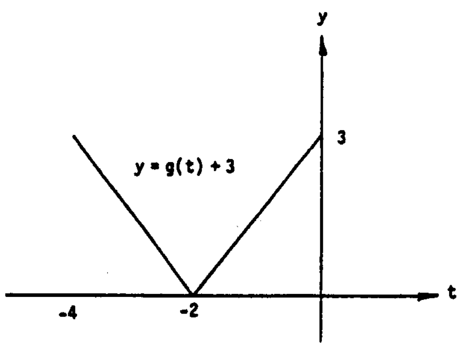
46. (a) domain: $[0,4]$; range: $[-3,0]$



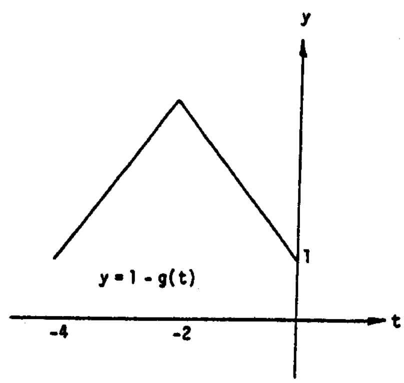
(b) domain: $[-4,0]$; range: $[0,3]$



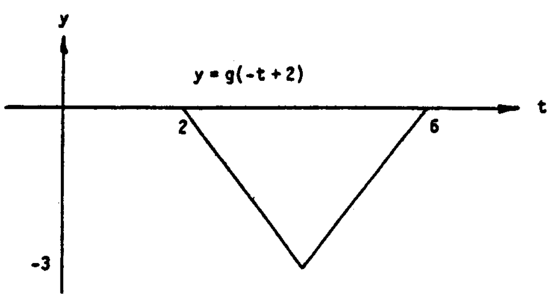
(c) domain: $[-4,0]$; range: $[0,3]$



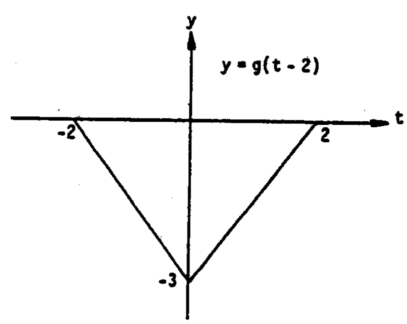
(d) domain: $[-4,0]$; range: $[1,4]$



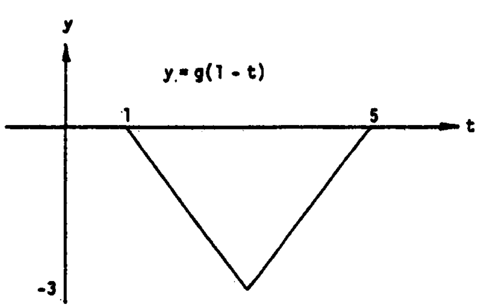
(e) domain: $[2,6]$; range: $[-3,0]$



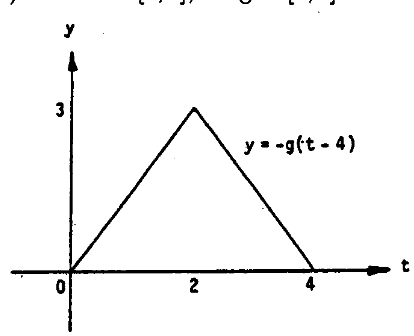
(f) domain: $[-2,2]$; range: $[-3,0]$



(g) domain: $[1,5]$; range: $[-3,0]$



(h) domain: $[0,4]$; range: $[0,3]$



47. (a) Because the circumference of the original circle was 8π and a piece of length x was removed.

(b) $r = \frac{8\pi - x}{2\pi} = 4 - \frac{x}{2\pi}$

(c) $h = \sqrt{16 - r^2} = \sqrt{16 - \left(4 - \frac{x}{2\pi}\right)} = \sqrt{16 - \left(16 - \frac{4x}{\pi} + \frac{x^2}{4\pi^2}\right)} = \sqrt{\frac{4x}{\pi} - \frac{x^2}{4\pi^2}} = \sqrt{\frac{16\pi x}{4\pi^2} - \frac{x^2}{4\pi^2}} = \frac{\sqrt{16\pi x - x^2}}{2\pi}$

(d) $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{8\pi - x}{2\pi}\right)^2 \cdot \frac{\sqrt{16\pi x - x^2}}{2\pi} = \frac{(8\pi - x)^2\sqrt{16\pi x - x^2}}{24\pi^2}$

48. (a) Note that 2 mi = 10,560 ft, so there are $\sqrt{800^2 + x^2}$ feet of river cable at \$180 per foot and (10,560 − x) feet of land cable at \$100 per foot. The cost is $C(x) = 180\sqrt{800^2 + x^2} + 100(10{,}560 - x)$

(b) C(0) = \$1,200,000
C(500) ≈ \$1,175,812
C(1000) ≈ \$1,186,512
C(1500) = \$1,212,000
C(2000) ≈ \$1,243,732
C(2500) ≈ \$1,278,479
C(3000) ≈ \$1,314,870
Values beyond this are all larger. It would appear that the least expensive location is less than 2000 ft from point P.

49. (a) Yes. Since $(f \cdot g)(-x) = f(-x) \cdot g(-x) = f(x) \cdot g(x) = (f \cdot g)(x)$, the function $(f \cdot g)(x)$ will also be even.

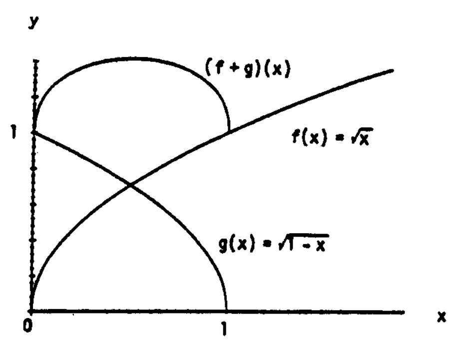
(b) The product will be even, since
$$\begin{aligned}(f \cdot g)(-x) &= f(-x) \cdot g(-x)\\ &= (-f(x)) \cdot (-g(x))\\ &= f(x) \cdot g(x)\\ &= (f \cdot g)(x).\end{aligned}$$

(c) Yes, $f(x) = 0$ is both even and odd since $f(-x) = -f(x) = f(x)$

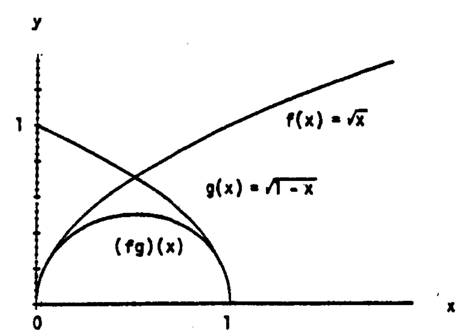
50. (a) Pick 11, for example: $11 + 5 = 16 \to 2 \cdot 16 = 32 \to 32 - 6 = 26 \to 26/2 = 13 \to 13 - 2 = 11$, the original number.

(b) $f(x) = \frac{2(x+5) - 6}{2} - 2 = x$, the number you started with.

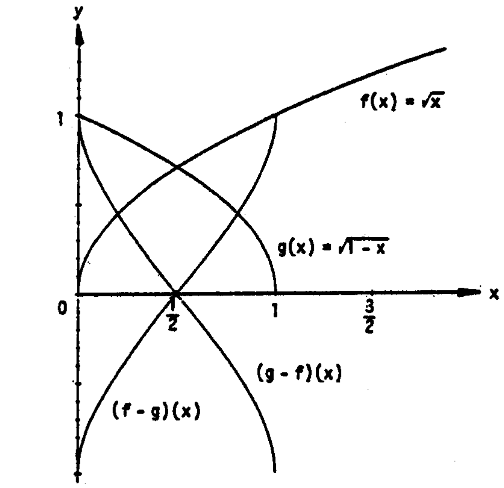
51. (a)



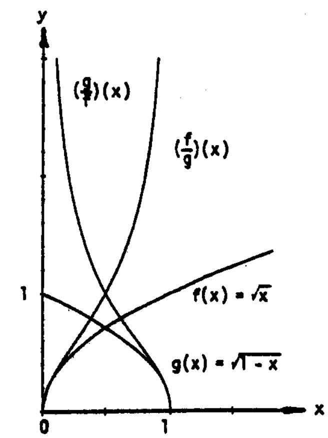
(b)



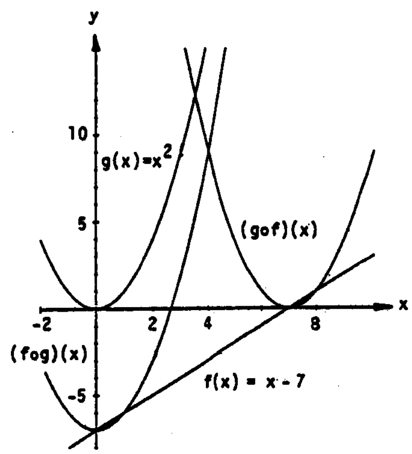
(c)



(d)

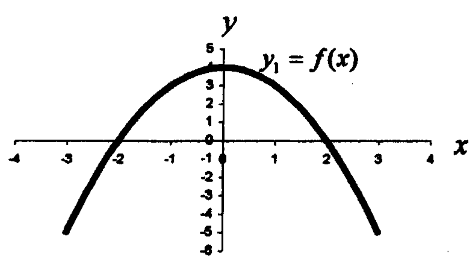


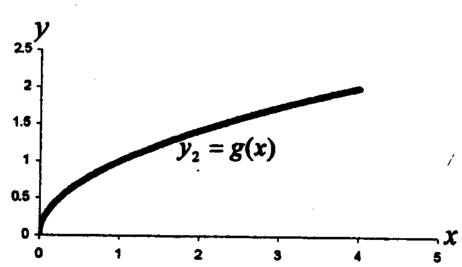
52.

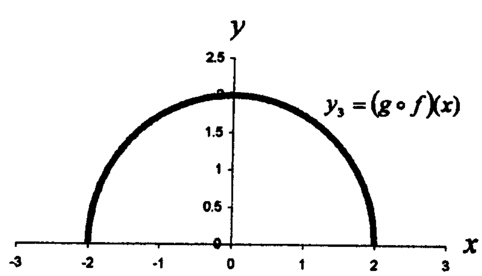


53. (a) $y_4 = (f \circ g)(x)$; $y_3 = (g \circ f)(x)$

(b)

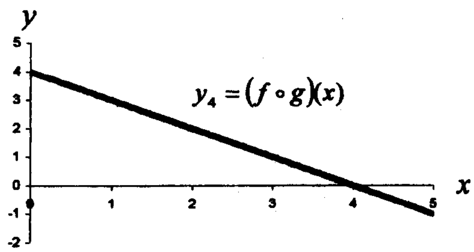






$D(g \circ f) = [-2, 2]$; The domain of $g \circ f$ is the set of all values of x in the domain of f for which the values $y_1 = f(x)$ are in the domain of g.
$R(g \circ f) = [0, 2]$; The range of $g \circ f$ is the subset of the range of g that includes all the values of g(x) evaluated at the values from the range of f where g(x) is defined.

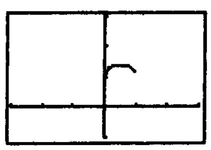
(c) The graphs of $y_1 = f(x)$ and $y_2 = g(x)$ are shown in part (a).



$D(f \circ g) = [0, \infty)$; The domain of $f \circ g$ is the set of all values of x in the domain of g for which the values $y_2 = g(x)$ are in the domain of f.
$R(f \circ g) = (-\infty, 4]$; The range of $f \circ g$ is the subset of the range of f that includes all the values of f(x) evaluated at the values from the range of g where f(x) is defined.

(d) $(g \circ f)(x) = \sqrt{4 - x^2}$; $D(g \circ f) = [-2, 2]$; $R(g \circ f) = [0, 2]$
$(f \circ g)(x) = 4 - (\sqrt{x})^2 = 4 - x$ for $x \geq 0$; $D(f \circ g) = [0, \infty)$; $R(f \circ g) = (-\infty, 4]$

54. (a)



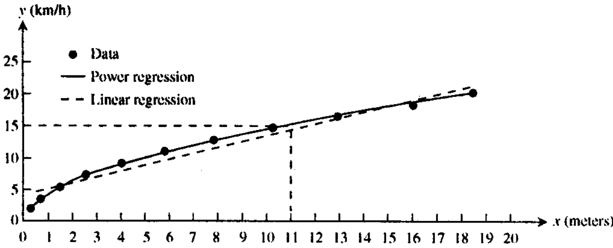
$[-3, 3]$ by $[-1, 3]$

(b) Domain of y_1: $[0, \infty)$
Domain of y_2: $(-\infty, 1]$
Domain of y_3: $[0, 1]$

(c) The functions $y_1 - y_2$, $y_2 - y_1$, and $y_1 \cdot y_2$ all have domain $[0, 1]$, the same as the domain of $y_1 + y_2$ found in part (b).
Domain of $\frac{y_1}{y_2}$: $[0, 1)$
Domain of $\frac{y_2}{y_1}$: $(0, 1]$

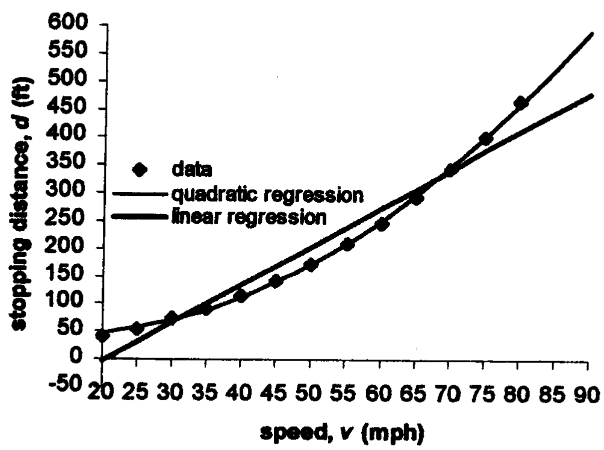
(d) The domain of a sum, difference, or product of two functions is the intersection of their domains.
The domain of a quotient of two functions is the intersection of their domains with any zeros of the denominator removed.

55. (a) The power regression function on the TI-92 Plus calculator gives $y = 4.44647x^{0.511414}$
(b)



(c) 15.2 km/h
(d) The linear regression function on the TI-92 Plus calculator gives $y = 0.913675x + 4.189976$ and it is shown on the graph in part (b). The linear regression function gives a speed of 14.2 km/h when y = 11m. The power regression curve in part (a) better fits the data.

56. (a) Let v represent the speed in miles per hour and d the stopping distance in feet. The quadratic regression function on the TI-92 Plus calculator gives $d = 0.0886v^2 - 1.97v + 50.1$.
(b)



(c) From the graph in part (b), the stopping distance is about 370 feet when the vehicle speed is 72 mph and it is about 525 feet when the speed is 85 mph.
Algebraically: $d_{quadratic}(72) = 0.0886(72)^2 - 1.97(72) + 50.1 = 367.6$ ft.

$d_{quadratic}(85) = 0.0886(85)^2 - 1.97(85) + 50.1 = 522.8$ ft.

(d) The linear regression function on the TI-92 Plus calculator gives $d = 6.89v - 140.4 \Rightarrow d_{linear}(72)$ $= 6.89(72) - 140.4 = 355.7$ ft and $d_{linear}(85) = 6.89(85) - 140.4 = 445.2$ ft. The linear regression line is shown on the graph in part (b). The quadratic regression curve clearly gives the better fit.

P.3 EXPONENTIAL FUNCTIONS

1. The graph of $y = 2^x$ is increasing from left to right and has the negative x-axis as an asymptote. (a)

2. The graph of $y = 3^{-x}$ or, equivalently, $y = \left(\frac{1}{3}\right)^x$, is decreasing from left to right and has the positive x-axis as an asymptote. (d)

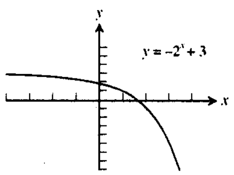
3. The graph of $y = -3^{-x}$ is the reflection about the x-axis of the graph in Exercise 2. (e)

4. The graph of $y = -0.5^{-x}$ or, equivalently, $y = -2^{x}$, is the reflection about the x-axis of the graph in Exercise 1. (c)

5. The graph of $y = 2^{-x} - 2$ is decreasing from left to right and has the line $y = -2$ as an asymptote. (b)

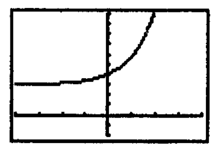
6. The graph of $y = 1.5^{x} - 2$ is increasing from left to right and has the line $y = -2$ as an asymptote. (f)

7.



Domain: $(-\infty, \infty)$
Range: $(-\infty, 3)$
x-intercept: ≈ 1.585
y-intercept: 2

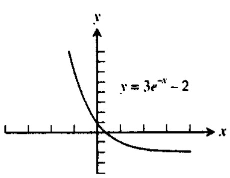
8.



[−4, 4] by [−2, 10]

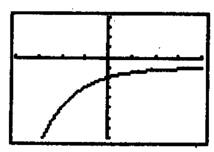
Domain: $(-\infty, \infty)$
Range: $(3, \infty)$
x-intercept: None
y-intercept: 4

9.



Domain: $(-\infty, \infty)$
Range: $(-2, \infty)$
x-intercept: ≈ 0.405
y-intercept: 1

10.



[−4, 4] by [−8, 4]

Domain: $(-\infty, \infty)$
Range: $(-\infty, -1)$
x-intercept: None
y-intercept: −2

11. $9^{2x} = (3^2)^{2x} = 3^{4x}$

12. $16^{3x} = (2^4)^{3x} = 2^{12x}$

13. $\left(\frac{1}{8}\right)^{2x} = (2^{-3})^{2x} = 2^{-6x}$

14. $\left(\frac{1}{27}\right)^{x} = (3^{-3})^{x} = 3^{-3x}$

15.

x	y	Δy
1	−1	
		2
2	1	
		2
3	3	
		2
4	5	

16.

x	y	Δy
1	1	
		−3
2	−2	
		−3
3	−5	
		−3
4	−8	

17.

x	y	Δy
1	1	
		3
2	4	
		5
3	9	
		7
4	16	

18.

x	y	ratio
1	8.155	
		2.718
2	22.167	
		2.718
3	60.257	
		2.718
4	163.79	

19. The slope of a straight line is $m = \frac{\Delta y}{\Delta x} \rightarrow \Delta y = m(\Delta x)$. In Exercise 15, each $\Delta x = 1$ and $m = 2 \rightarrow$ each $\Delta y = 2$, and in problem 16, each $\Delta x = 1$ and $m = -3 \rightarrow$ each $\Delta y = -3$. If the changes in x are constant for a linear function, say $\Delta x = c$, then the changes in y are also constant, specifically, $\Delta y = mc$.

20. From the table in Exercise 17, it can be seen that $\Delta y = 2x + 1$. Some examples are: $\Delta y = 9 - 4 = 5 = 2(2) + 1 = 2x + 1$ and $\Delta y = 16 - 9 = 7 = 2(3) + 1 = 2x + 1$. As x changes from $x = 1000$ to $x = 1001$, the change in y is $\Delta y = 2(1000) + 1 = 2001$. As x changes from n to $n + 1$, where n is an arbitrary positive integer, the change in y is $\Delta y = 2n + 1$.

21. Since $f(1) = 4.5$ we have $ka = 4.5$, and since $f(-1) = 0.5$ we have $ka^{-1} = 0.5$.
Dividing, we have

$\frac{ka}{ka^{-1}} = \frac{4.5}{0.5}$

$a^2 = 9$

$a = \pm 3$

Since $f(x) = k \cdot a^x$ is an exponential function, we require $a > 0$, so $a = 3$. Then $ka = 4.5$ gives $3k = 4.5$, so $k = 1.5$. The values are $a = 3$ and $k = 1.5$.

22. Since $f(1) = 1.5$ we have $ka = 1.5$, and since $f(-1) = 6$ we have $ka^{-1} = 6$.
Dividing, we have

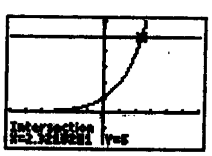
$\frac{ka}{ka^{-1}} = \frac{1.5}{6}$

$a^2 = 0.25$

$a = \pm 0.5$

Since $f(x) = k \cdot a^x$ is an exponential function, we require $a > 0$, so $a = 0.5$. Then $ka = 1.5$ gives $0.5k = 1.5$, so $k = 3$. The values are $a = 0.5$ and $k = 3$.

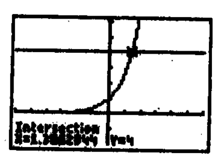
23.



[−6, 6] by [−2, 6]

$x \approx 2.3219$

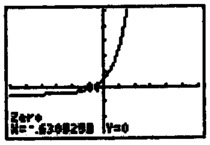
24.



[−6, 6] by [−2, 6]

$x \approx 1.3863$

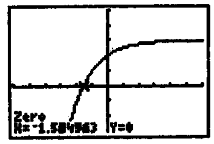
25.



[−6, 6] by [−3, 5]

$x \approx -0.6309$

26.



[−6, 6] by [−3, 5]

$x \approx -1.5850$

27. $5422(1.018)^{19} \approx 7609.7$ million

28. (a) When $t = 0$, $B = 100e^0 = 100$. There were 100 bacteria present initially.
    (b) When $t = 6$, $B = 100e^{0.693(6)} \approx 6394.351$. After 6 hours, there are about 6394 bacteria.
    (c) Solving $100e^{0.693t} = 200$ graphically, we find that $t \approx 1.000$. The population will be 200 after about 1 hour. Since the population doubles (from 100 to 200) in about 1 hour, the doubling time is about 1 hour.

29. Let t be the number of years. Solving $500{,}000(1.0375)^t = 1{,}000{,}000$ graphically, we find that $t \approx 18.828$. The population will reach 1 million in about 19 years.

30. (a) The population is given by $P(t) = 6250(1.0275)^t$, where t is the number of years after 1890.
    Population in 1915: $P(25) \approx 12{,}315$
    Population in 1940: $P(50) \approx 24{,}265$
    (b) Solving $P(t) = 50{,}000$ graphically, we find that $t \approx 76.651$. The population reached 50,000 about 77 years after 1890, in 1967.

31. (a) $A(t) = 6.6\left(\frac{1}{2}\right)^{t/14}$
    (b) Solving $A(t) = 1$ graphically, we find that $t \approx 38$. There will be 1 gram remaining after about 38.1145 days.

32. Let t be the number of years. Solving $2300(1.06)^t = 4150$ graphically, we find that $t \approx 10.129$. It will take about 10.129 years. (If the interest is not credited to the account until the end of each year, it will take 11 years.)

33. Let A be the amount of the initial investment, and let t be the number of years. We wish to solve $A(1.0625)^t = 2A$, which is equivalent to $1.0625^t = 2$. Solving graphically, we find that $t \approx 11.433$. It will take about 11.433 years. (If the interest is credited at the end of each year, it will take 12 years.)

34. Let A be the amount of the initial investment, and let t be the number of years. We wish to solve $A\left(1 + \frac{0.0625}{12}\right)^{12t} = 2A$, which is equivalent to $\left(1 + \frac{0.0625}{12}\right)^{12t} = 2$. Solving graphically, we find that $t \approx 11.119$. It will take about 11.119 years. (If the interest is credited at the end of each month, it will take 11 years 2 months.)

35. Let A be the amount of the initial investment, and let t be the number of years. We wish to solve $Ae^{0.0625t} = 2A$, which is equivalent to $e^{0.0625t} = 2$. Solving graphically, we find that $t \approx 11.090$. It will take about 11.090 years.

36. Let A be the amount of the initial investment, and let t be the number of years. We wish to solve $A(1.0575)^t = 3A$, which is equivalent to $1.0575^t = 3$. Solving graphically, we find that $t \approx 19.650$. It will take about 19.650 years. (If the interest is credited at the end of each year, it will take 20 years.)

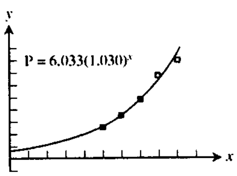
37. Let A be the amount of the initial investment, and let t be the number of years. We wish to solve $A\left(1 + \frac{0.0575}{365}\right)^{365t} = 3A$, which is equivalent to $\left(1 + \frac{0.0575}{365}\right)^{365t} = 3$. Solving graphically, we find that $t \approx 19.108$. It will take about 19.108 years.

38. Let A be the amount of the initial investment, and let t be the number of years. We wish to solve $Ae^{0.0575t} = 3A$, which is equivalent to $e^{0.0575t} = 3$. Solving graphically, we find that $t \approx 19.106$. It will take about 19.106 years.

39. After t hours, the population is $P(t) = 2^{t/0.5}$ or, equivalently, $P(t) = 2^{2t}$. After 24 hours, the population is $P(24) = 2^{48} \approx 2.815 \times 10^{14}$ bacteria.

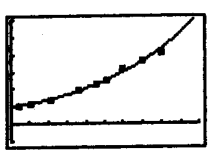
40. (a) Each year, the number of cases is $100\% - 20\% = 80\%$ of the previous year's number of cases. After t years, the number of cases will be $C(t) = 10{,}000(0.8)^t$. Solving $C(t) = 1000$ graphically, we find that $t \approx 10.319$. It will take about 10.319 years.
(b) Solving $C(t) = 1$ graphically, we find that $t \approx 41.275$. It will take about 41.275 years.

41. (a) Let $x = 0$ represent 1900, $x = 1$ represent 1901, and so on. The regression equation is $P(x) = 6.033(1.030)^x$.



(b) The regression equation gives an estimate of $P(0) \approx 6.03$ million, which is not very close to the actual population.
(c) Since the equation is of the form $P(x) = P(0) \cdot 1.030^x$, the annual rate of growth is about 3%.

42. (a) The regression equation is $P(x) = 4.831(1.019)^x$.



[0, 100] by [−5, 30]

(b) $P(90) \approx 26.3$ million
(c) Since the equation is of the form $P(x) = P(0) \cdot 1.019^x$, the annual rate of growth is approximately 1.9%.

P.4 FUNCTIONS AND LOGARITHMS

1. Yes one-to-one, the graph passes the horizontal test.

2. Not one-to-one, the graph fails the horizontal test.

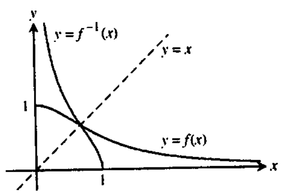
3. Not one-to-one since (for example) the horizontal line $y = 2$ intersects the graph twice.

4. Not one-to-one, the graph fails the horizontal test

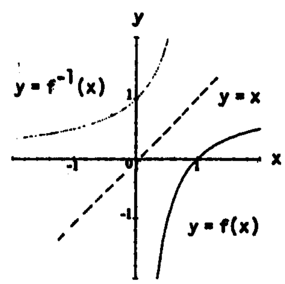
5. Yes one-to-one, the graph passes the horizontal test

6. Yes one-to-one, the graph passes the horizontal test

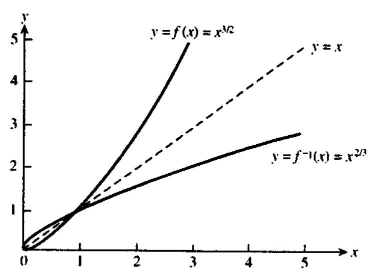
7. Domain: $0 < x \leq 1$, Range: $y \geq 0$



8. Domain: $x < 1$, Range: $y > 0$

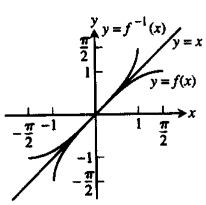


9.



Domain: $x \geq 0$, Range: $y \geq 0$

10. Domain: $-1 \leq x \leq 1$, Range: $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$



11. Step 1: $y = x^2 + 1 \Rightarrow x^2 = y - 1 \Rightarrow x = \sqrt{y-1}$
Step 2: $y = \sqrt{x-1} = f^{-1}(x)$

12. Step 1: $y = x^2 \Rightarrow x = -\sqrt{y}$
Step 2: $y = -\sqrt{x} = f^{-1}(x)$

13. Step 1: $y = x^3 - 1 \Rightarrow x^3 = y + 1 \Rightarrow x = (y+1)^{1/3}$
Step 2: $y = \sqrt[3]{x+1} = f^{-1}(x)$

14. Step 1: $y = x^2 - 2x + 1 \Rightarrow y = (x-1)^2 \Rightarrow \sqrt{y} = x - 1 \Rightarrow x = \sqrt{y} + 1$
Step 2: $y = 1 + \sqrt{x} = f^{-1}(x)$

15. Step 1: $y = (x+1)^2 \Rightarrow \sqrt{y} = x + 1 \Rightarrow x = \sqrt{y} - 1$
Step 2: $y = \sqrt{x} - 1 = f^{-1}(x)$

16. Step 1: $y = x^{2/3} \Rightarrow x = y^{3/2}$
Step 2: $y = x^{3/2} = f^{-1}(x)$

17. $y = 2x + 3 \to y - 3 = 2x \to \frac{y-3}{2} = x$. Interchange x and y: $\frac{x-3}{2} = y \to f^{-1}(x) = \frac{x-3}{2}$
Verify.
$(f \circ f^{-1})(x) = f\left(\frac{x-3}{2}\right) = 2\left(\frac{x-3}{2}\right) + 3 = (x-3) + 3 = x$
$(f^{-1} \circ f)(x) = f^{-1}(2x+3) = \frac{(2x+3)-3}{2} = \frac{2x}{2} = x$

18. $y = 5 - 4x \to 4x = 5 - y \to x = \frac{5-y}{4}$. Interchange x and y: $y = \frac{5-x}{4} \to f^{-1}(x) = \frac{5-x}{4}$
Verify.
$(f \circ f^{-1})(x) = f\left(\frac{5-x}{4}\right) = 5 - 4\left(\frac{5-x}{4}\right) = 5 - (5-x) = x$
$(f^{-1} \circ f)(x) = f^{-1}(5-4x) = \frac{5-(5-4x)}{4} = \frac{4x}{4} = x$

19. $y = x^3 - 1 \to y + 1 = x^3 \to (y+1)^{1/3} = x$. Interchange x and y: $(x+1)^{1/3} = y$
$\to f^{-1}(x) = (x+1)^{1/3}$ or $\sqrt[3]{x+1}$
Verify.
$(f \circ f^{-1})(x) = f(\sqrt[3]{x+1}) = (\sqrt[3]{x+1})^3 - 1 = (x+1) - 1 = x$
$(f^{-1} \circ f)(x) = f^{-1}(x^3 - 1) = \sqrt[3]{(x^3-1)+1} = \sqrt[3]{x^3} = x$

20. $y = x^2 + 1,\ x \ge 0 \to y - 1 = x^2,\ x \ge 0 \to \sqrt{y-1} = x$.
Interchange x and y: $\sqrt{x-1} = y \to f^{-1}(x) = \sqrt{x-1}$ or $(x-1)^{1/2}$
Verify. For $x \ge 1$ (the domain of f^{-1}),
$(f \circ f^{-1})(x) = f(\sqrt{x-1}) = (\sqrt{x-1})^2 + 1 = (x-1) + 1 = x$
For $x > 0$ (the domain of f),
$(f^{-1} \circ f)(x) = f^{-1}(x^2+1) = \sqrt{(x^2+1)-1} = \sqrt{x^2} = |x| = x$

21. $y = x^2,\ x \le 0 \to x = -\sqrt{y}$. Interchange x and y: $y = -\sqrt{x} \to f^{-1}(x) = -\sqrt{x}$ or $-x^{1/2}$

Verify.

For $x \ge 0$ (the domain of f^{-1}), $(f \circ f^{-1})(x) = f(-\sqrt{x}) = (-\sqrt{x})^2 = x$

For $x \le 0$ (the domain of f), $(f^{-1} \circ f)(x) = f^{-1}(x^2) = -\sqrt{x^2} = -|x| = x$

22. $y = x^{2/3},\ x \ge 0 \to y^{3/2} = \left(x^{2/3}\right)^{3/2},\ x \ge 0 \to y^{3/2} = x$

Interchange x and y: $x^{3/2} = y \to f^{-1}(x) = x^{3/2}$

Verify.

For $x \ge 0$ (the domain of f^{-1}), $(f \circ f^{-1})(x) = f\left(x^{3/2}\right) = \left(x^{3/2}\right)^{2/3} = x$

For $x \ge 0$ (the domain of f), $(f^{-1} \circ f)(x) = f^{-1}\left(x^{2/3}\right) = \left(x^{2/3}\right)^{3/2} = |x| = x$

23. $y = -(x-2)^2,\ x \le 2 \to (x-2)^2 = -y,\ x \le 2 \to x - 2 = -\sqrt{-y} \to x = 2 - \sqrt{-y}$.

Interchange x and y: $y = 2 - \sqrt{-x} \to f^{-1}(x) = 2 - \sqrt{-x}$ or $2 - (-x)^{1/2}$

Verify.

For $x \le 0$ (the domain of f^{-1})

$(f \circ f^{-1})(x) = f(2 - \sqrt{-x}) = -[(2 - \sqrt{-x}) - 2]^2 = -(-\sqrt{-x})^2 = -|x| = x$

For $x \le 2$ (the domain of f),

$(f^{-1} \circ f)(x) = f^{-1}\left(-(x-2)^2\right) = 2 - \sqrt{(x-2)^2} = 2 - |x-2| = 2 + (x-2) = x$

24. $y = (x^2 + 2x + 1),\ x \ge -1 \to y = (x+1)^2,\ x \ge -1 \to \sqrt{y} = x + 1 \to \sqrt{y} - 1 = x$.

Interchange x and y: $\sqrt{x} - 1 = y \to f^{-1}(x) = \sqrt{x} - 1$ or $x^{1/2} - 1$

Verify.

For $x \ge 0$ (the domain of f^{-1}),

$(f \circ f^{-1})(x) = f(\sqrt{x} - 1) = \left[(\sqrt{x} - 1)^2 + 2(\sqrt{x} - 1) + 1\right] = (\sqrt{x})^2 - 2\sqrt{x} + 1 + 2\sqrt{x} - 2 + 1 = (\sqrt{x})^2 = x$

For $x \ge -1$ (the domain of f),

$(f^{-1} \circ f)(x) = f^{-1}(x^2 + 2x + 1) = \sqrt{x^2 + 2x + 1} - 1 = \sqrt{(x+1)^2} - 1 = |x+1| - 1 = (x+1) - 1 = x$

25. $y = \frac{1}{x^2},\ x > 0 \to x^2 = \frac{1}{y},\ x > 0 \to x = \sqrt{\frac{1}{y}} = \frac{1}{\sqrt{y}}$.

Interchange x and y: $y = \frac{1}{\sqrt{x}} \to f^{-1}(x) = \frac{1}{\sqrt{x}}$ or $\frac{1}{x^{1/2}}$

Verify.

For $x > 0$ (the domain of f^{-1}), $(f \circ f^{-1})(x) = f\left(\frac{1}{\sqrt{x}}\right) = \frac{1}{(1/\sqrt{x})^2} = x$

For $x > 0$ (the domain of f), $(f^{-1} \circ f)(x) = f^{-1}\left(\frac{1}{x^2}\right) = \frac{1}{\sqrt{1/x^2}} = \sqrt{x^2} = |x| = x$

26. $y = \frac{1}{x^3} \to x^3 = \frac{1}{y} \to x = \sqrt[3]{\frac{1}{y}} = \frac{1}{\sqrt[3]{y}}$.

Interchange x and y: $y = \frac{1}{\sqrt[3]{x}} \to f^{-1}(x) = \frac{1}{\sqrt[3]{x}}$ or $\frac{1}{x^{1/3}}$

Verify.

$$(f \circ f^{-1})(x) = f\left(\frac{1}{\sqrt[3]{x}}\right) = \frac{1}{\left(1/\sqrt[3]{x}\right)^3} = x$$

$$(f^{-1} \circ f)(x) = f^{-1}\left(\frac{1}{x^3}\right) = \frac{1}{\sqrt[3]{1/x^3}} = x$$

27. $y = \frac{2x+1}{x+3} \to xy + 3y = 2x + 1 \to xy - 2x = 1 - 3y \to (y-2)x = 1 - 3y \to x = \frac{1-3y}{y-2}$.

Interchange x and y: $y = \frac{1-3x}{x-2} \to f^{-1}(x) = \frac{1-3x}{x-2}$

Verify.

$$(f \circ f^{-1})(x) = f\left(\frac{1-3x}{x-2}\right) = \frac{2\left(\frac{1-3x}{x-2}\right)+1}{\frac{1-3x}{x-2}+3} = \frac{2(1-3x)+(x-2)}{(1-3x)+3(x-2)} = \frac{-5x}{-5} = x$$

$$(f^{-1} \circ f)(x) = f^{-1}\left(\frac{2x+1}{x+3}\right) = \frac{1-3\left(\frac{2x+1}{x+3}\right)}{\frac{2x+1}{x+3}-2} = \frac{(x+3)-3(2x+1)}{(2x+1)-2(x+3)} = \frac{-5x}{-5} = x$$

28. $y = \frac{x+3}{x-2} \to xy - 2y = x + 3 \to xy - x = 2y + 3 \to x(y-1) = 2y + 3 \to x = \frac{2y+3}{y-1}$.

Interchange x and y: $y = \frac{2x+3}{x-1} \to f^{-1}(x) = \frac{2x+3}{x-1}$

Verify.

$$(f \circ f^{-1})(x) = f\left(\frac{2x+3}{x-1}\right) = \frac{\frac{2x+3}{x-1}+3}{\frac{2x+3}{x-1}-2} = \frac{(2x+3)+3(x-1)}{(2x+3)-2(x-1)} = \frac{5x}{5} = x$$

$$(f^{-1} \circ f)(x) = f^{-1}\left(\frac{x+3}{x-2}\right) = \frac{2\left(\frac{x+3}{x-2}\right)+3}{\frac{x+3}{x-2}-1} = \frac{2(x+3)+3(x-2)}{(x+3)-(x-2)} = \frac{5x}{5} = x$$

29. $y = \left(e^a\right)^x - 1 \to e^a = 3 \to a = \ln 3 \to y = e^{x \ln 3} - 1$

(a) $D = (-\infty, \infty)$

(b) $R = (-1, \infty)$

30. $y = \left(e^a\right)^{x+1} \to e^a = 4 \to a = \ln 4 \to y = e^{(x+1) \ln 4} = e^{x \ln 4} e^{\ln 4} = 4e^{x \ln 4}$

(a) $D = (-\infty, \infty)$

(b) $R = (0, \infty)$

31. $y = 1 - (\ln 3) \log_3 x = 1 - (\ln 3) \frac{\ln x}{\ln 3} = 1 - \ln x$

(a) $D = (0, \infty)$

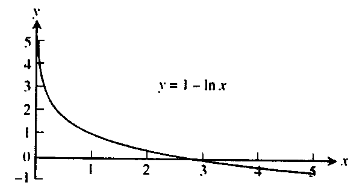
(b) $R = (-\infty, \infty)$

32. $y = (\ln 10) \log (x+2) = (\ln 10) \frac{\ln (x+2)}{\ln 10} = \ln (x+2)$

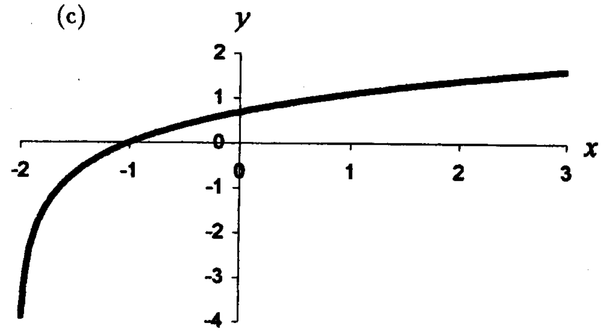
(a) $D = (-2, \infty)$

(b) $R = (-\infty, \infty)$

(c)



(c)



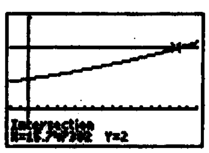
33. $(1.045)^t = 2$

$\ln (1.045)^t = \ln 2$

$t \ln 1.04 = \ln 2$

$t = \frac{\ln 2}{\ln 1.045} \approx 15.75$

Graphical support:



[−2, 18] by [−1, 3]

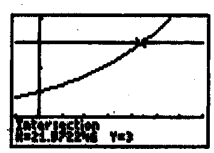
34. $e^{0.05t} = 3$

$\ln e^{0.05t} = \ln 3$

$0.05t = \ln 3$

$t = \frac{\ln 3}{0.05} = 20 \ln 3 \approx 21.97$

Graphical support:



[−5, 35] by [−1, 4]

35. $e^x + e^{-x} = 3$

$e^x - 3 + e^{-x} = 0$

$e^x(e^x - 3 + e^{-x}) = e^x(0)$

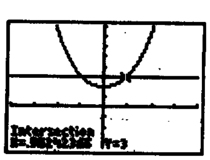
$(e^x)^2 - 3e^x + 1 = 0$

$$e^x = \frac{3 \pm \sqrt{(-3)^2 - 4(1)(1)}}{2(1)}$$

$$e^x = \frac{3 \pm \sqrt{5}}{2}$$

$$x = \ln\left(\frac{3 \pm \sqrt{5}}{2}\right) \approx -0.96 \text{ or } 0.96$$

Graphical support:



[−4, 4] by [−4, 8]

36. $2^x + 2^{-x} = 5$

$2^x - 5 + 2^{-x} = 0$

$2^x(2^x - 5 + 2^{-x}) = 2^x(0)$

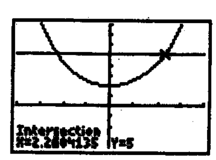
$(2^x)^2 - 5(2^x) + 1 = 0$

$$2^x = \frac{5 \pm \sqrt{(-5)^2 - 4(1)(1)}}{2(1)}$$

$$2^x = \frac{5 \pm \sqrt{21}}{2}$$

$$x = \log_2\left(\frac{5 \pm \sqrt{21}}{2}\right) \approx -2.26 \text{ or } 2.26$$

Graphical support:



[−4, 4] by [−4, 8]

37. $\ln y = 2t + 4 \rightarrow e^{\ln y} = e^{2t+4} \rightarrow y = e^{2t+4}$

38. $\ln(y-1) - \ln 2 = x + \ln x \rightarrow \ln(y-1) = x + \ln x + \ln 2 \rightarrow e^{\ln(y-1)} = e^{x + \ln x + \ln 2} \rightarrow y - 1 = e^x(x)(2)$
$\rightarrow y = 2xe^x + 1$

39. (a) $y = \frac{100}{1+2^{-x}} \rightarrow 1 + 2^{-x} = \frac{100}{y} \rightarrow 2^{-x} = \frac{100}{y} - 1 \rightarrow \log_2(2^{-x}) = \log_2\left(\frac{100}{y} - 1\right) \rightarrow -x = \log_2\left(\frac{100}{y} - 1\right)$

$\rightarrow x = -\log_2\left(\frac{100}{y} - 1\right) = -\log_2\left(\frac{100-y}{y}\right) = \log_2\left(\frac{y}{100-y}\right).$

Interchange x and y: $y = \log_2\left(\frac{x}{100-x}\right) \rightarrow f^{-1}(x) = \log_2\left(\frac{x}{100-x}\right)$

Verify.

$$(f \circ f^{-1})(x) = f\left(\log_2\left(\frac{x}{100-x}\right)\right) = \frac{100}{1+2^{-\log_2\left(\frac{x}{100-x}\right)}} = \frac{100}{1+2^{\log_2\left(\frac{100-x}{x}\right)}} = \frac{100}{1+\frac{100-x}{x}}$$

$$= \frac{100x}{x+(100-x)} = \frac{100x}{100} = x$$

$$(f^{-1} \circ f)(x) = f^{-1}\left(\frac{100}{1+2^{-x}}\right) = \log_2\left(\frac{\frac{100}{1+2^{-x}}}{100 - \frac{100}{1+2^{-x}}}\right) = \log_2\left(\frac{100}{100(1+2^{-x}) - 100}\right)$$

$$= \log_2\left(\frac{1}{2^{-x}}\right) = \log_2(2^x) = x$$

(b) $y = \frac{50}{1+1.1^{-x}} \rightarrow 1 + 1.1^{-x} = \frac{50}{y} \rightarrow 1.1^{-x} = \frac{50}{y} - 1 \rightarrow \log_{1.1}(1.1^{-x}) = \log_{1.1}\left(\frac{50}{y} - 1\right) \rightarrow -x = \log_{1.1}\left(\frac{50}{y} - 1\right)$

$\rightarrow x = -\log_{1.1}\left(\frac{50}{y} - 1\right) = -\log_{1.1}\left(\frac{50-y}{y}\right) = \log_{1.1}\left(\frac{y}{50-y}\right).$

Interchange x and y: $y = \log_{1.1}\left(\frac{x}{50-x}\right) \rightarrow f^{-1}(x) = \log_{1.1}\left(\frac{x}{50-x}\right)$

Verify.

$$(f \circ f^{-1})(x) = f\left(\log_{1.1}\left(\frac{x}{50-x}\right)\right) = \frac{50}{1+1.1^{-\log_{1.1}\left(\frac{x}{50-x}\right)}} = \frac{50}{1+1.1^{\log_{1.1}\left(\frac{50-x}{x}\right)}} = \frac{50}{1+\frac{50-x}{x}}$$

$$= \frac{50x}{x+(50-x)} = \frac{50x}{50} = x$$

$$(f^{-1} \circ f)(x) = f^{-1}\left(\frac{50}{1+1.1^{-x}}\right) = \log_{1.1}\left(\frac{\frac{50}{1+1.1^{-x}}}{50 - \frac{50}{1+1.1^{-x}}}\right) = \log_{1.1}\left(\frac{50}{50(1+1.1^{-x}) - 50}\right)$$

$$= \log_{1.1}\left(\frac{1}{1.1^{-x}}\right) = \log_{1.1}(1.1^x) = x$$

40. (a) Suppose that $f(x_1) = f(x_2)$. Then $mx_1 + b = mx_2 + b$ so $mx_1 = mx_2$. Since $m \neq 0$, this gives $x_1 = x_2$.

(b) $y = mx + b \rightarrow y - b = mx \rightarrow \frac{y-b}{m} = x.$

Interchange x and y: $\frac{x-b}{m} = y \rightarrow f^{-1}(x) = \frac{x-b}{m}$

The slopes are reciprocals.

(c) If the original functions both have slope m, each of the inverse functions will have slope $\frac{1}{m}$. The graphs of the inverses will be parallel lines with nonzero slope.

(d) If the original functions have slopes m and $-\frac{1}{m}$, respectively, then the inverse functions will have slopes $\frac{1}{m}$ and $-m$, respectively. Since each of $\frac{1}{m}$ and $-m$ is the negative reciprocal of the other, the graphs of the inverses will be perpendicular lines with nonzero slopes.

41. (a) Amount $= 8\left(\frac{1}{2}\right)^{t/12}$

(b) $8\left(\frac{1}{2}\right)^{t/12} = 1 \rightarrow \left(\frac{1}{2}\right)^{t/12} = \frac{1}{8} \rightarrow \left(\frac{1}{2}\right)^{t/12} = \left(\frac{1}{2}\right)^{3} \rightarrow \frac{t}{12} = 3 \rightarrow t = 36$

There will be 1 gram remaining after 36 hours.

42. $500(1.0475)^t = 1000 \rightarrow 1.0475^t = 2 \rightarrow \ln(1.0475^t) = \ln 2 \rightarrow t \ln 1.0475 = \ln 2 \rightarrow t = \frac{\ln 2}{\ln 1.0475} \approx 14.936$

It will take about 14.936 years. (If the interest is paid at the end of each year, it will take 15 years.)

43. $375{,}000(1.0225)^t = 1{,}000{,}000 \rightarrow 1.0225^t = \frac{8}{3} \rightarrow \ln(1.0225^t) = \ln\left(\frac{8}{3}\right) \rightarrow t \ln 1.0225 = \ln\left(\frac{8}{3}\right)$

$\rightarrow t = \frac{\ln(8/3)}{\ln 1.0225} \approx 44.081$

It will take about 44.081 years.

44. Let O = original sound level $= 10 \log_{10}(I \times 10^{12})$ db from Equation (1) in the text. Solving $O + 10 = 10 \log_{10}(kI \times 10^{12})$ for $k \Rightarrow 10 \log_{10}(I \times 10^{12}) + 10 = 10 \log_{10}(kI \times 10^{12})$

$\Rightarrow \log_{10}(I \times 10^{12}) + 1 = \log_{10}(kI \times 10^{12}) \Rightarrow \log_{10}(I \times 10^{12}) + 1 = \log_{10} k + \log_{10}(I \times 10^{12})$

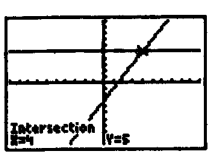
$\Rightarrow 1 = \log_{10} k \Rightarrow 1 = \frac{\ln k}{\ln 10} \Rightarrow \ln k = \ln 10 \Rightarrow k = 10$

45. Sound level with intensity = 10I is $10 \log_{10}(10I \times 10^{12}) = 10\left[\log_{10} 10 + \log_{10}(I \times 10^{12})\right]$

$= 10 + 10 \log_{10}(I \times 10^{12}) =$ original sound level $+ 10 \Rightarrow$ an increase of 10 db

46. $y = y_0 e^{-0.18t}$ represents the decay equation; solving $(0.9)y_0 = y_0 e^{-0.18t} \Rightarrow t = \frac{\ln(0.9)}{-0.18} \approx 0.585$ days

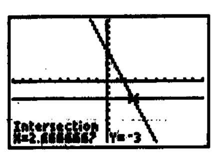
47.



[−10, 10] by [−10, 10]

(4, 5)

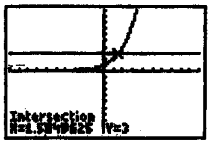
48.



[−10, 10] by [−10, 10]

$\left(\frac{8}{3}, -3\right) \approx (2.67, -3)$

49. (a)

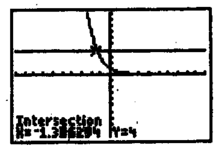


[−10, 10] by [−10, 10]

(1.58, 3)

(b) No points of intersection, since $2^x > 0$ for all values of x.

50. (a)

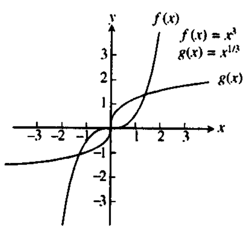


[−10, 10] by [−10, 10]

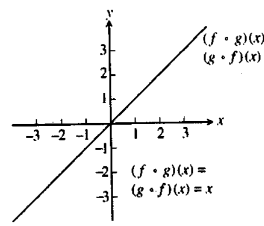
(−1.39, 4)

(b) No points of intersection, since $e^{-x} > 0$ for all values of x.

51. (a)

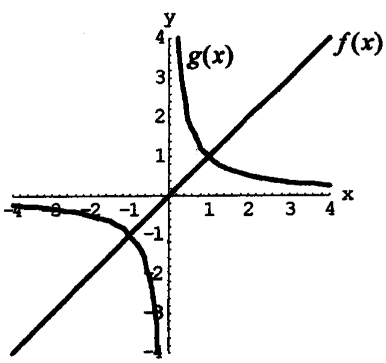


(b) and (c)

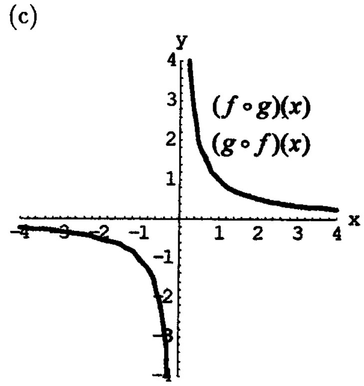


We conclude that f and g are inverses of each other because $(f \circ g)(x) = (g \circ f)(x) = x$, the identity function.

52. (a)

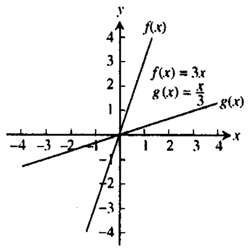


(b) and (c)

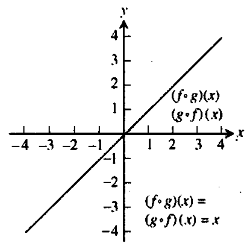


We conclude that f is the identity function because $(f \circ g)(x) = (g \circ f)(x) = \frac{1}{x} = g(x)$

53. (a)

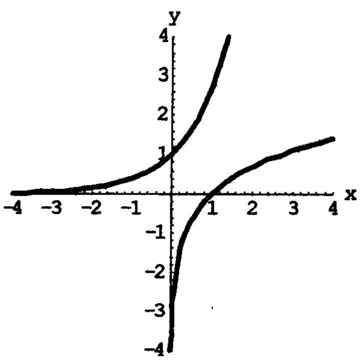


(b) and (c)

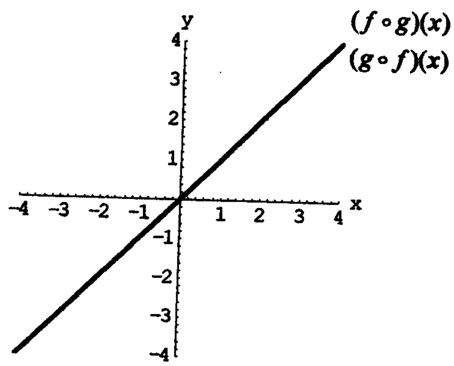


We conclude that f and g are inverses of each other because $(f \circ g)(x) = (g \circ f)(x) = x$, the identity function.

54. (a)

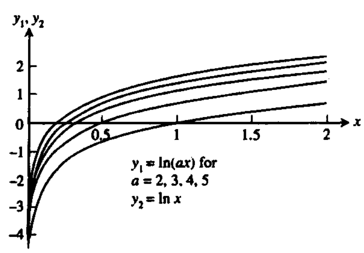


(b) and (c)

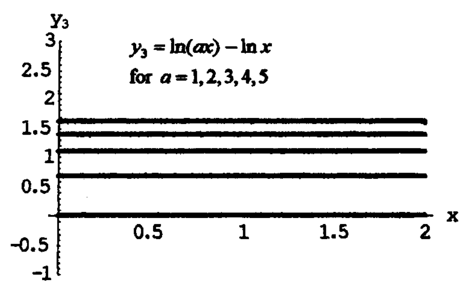


We conclude that f and g are inverses of each other because $(f \circ g)(x) = (g \circ f)(x) = x$, the identity function.

55. (a)



(b)



The graphs of y_1 appear to be vertical translates of y_2

The graphs of $y_1 - y_2$ support the finding in part (a).

(c) $y_3 = y_1 - y_2 = \ln ax - \ln x = (\ln a + \ln x) - \ln x = \ln a$, a constant.

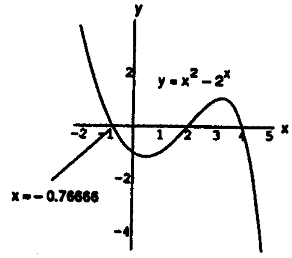
56. (a) y_2 is a vertical shift (upward) of y_1, although it's difficult to see that near the vertical asymptote at $x = 0$. One might use "trace" or "table" to verify this.

(b) Each graph of y_3 is a horizontal line.

(c) The graphs of y_4 and $y = a$ are the same.

(d) $e^{y_2 - y_1} = a$, $\ln(e^{y_2 - y_1}) = \ln a$, $y_2 - y_1 = \ln a$, $y_1 = y_2 - \ln a = \ln x - \ln a$

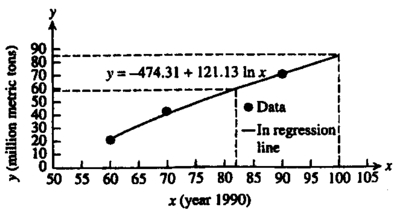
57. From zooming in on the graph at the right, we estimate the third root to be $x \approx -0.76666$



58. The functions $f(x) = x^{\ln 2}$ and $g(x) = 2^{\ln x}$ appear to have identical graphs for $x > 0$. This is no accident, because $x^{\ln 2} = e^{\ln 2 \cdot \ln x} = (e^{\ln 2})^{\ln x} = 2^{\ln x}$.

59. (a) The LnReg command on the TI-92 Plus calculator gives $y(x) = -474.31 + 121.13 \ln x$ $\Rightarrow y(82) = -474.31 + 121.13 \ln(82) = 59.48$ million metric tons produced in 1982 and $y(100) = -474.31 + 121.13 \ln(100) = 83.51$ million metric tons produced in 2000.

(b)



(c) From the graph in part (b), $y(82) \approx 59$ and $y(100) \approx 84$.

60. (a) $y = -2539.852 + 636.896 \ln x$

(b) When $x = 75$, $y \approx 209.94$. About 209.94 million metric tons were produced.

(c) $-2539.852 + 636.896 \ln x = 400$

$$636.896 \ln x = 2939.852$$

$$\ln x = \frac{2939.852}{636.896}$$

$$x = e^{\frac{2939.852}{636.896}} \approx 101.08$$

According to the regression equation, Saudi Arabian oil production will reach 400 million metric tons when $x \approx 101.08$, in about 2001.

P.5 TRIGONOMETRIC FUNCTIONS AND THEIR INVERSES

1. (a) $s = r\theta = (10)\left(\frac{4\pi}{5}\right) = 8\pi$ m

   (b) $s = r\theta = (10)(110^\circ)\left(\frac{\pi}{180^\circ}\right) = \frac{110\pi}{18} = \frac{55\pi}{9}$ m

2. $\theta = \frac{s}{r} = \frac{10\pi}{8} = \frac{5\pi}{4}$ radians and $\frac{5\pi}{4}\left(\frac{180^\circ}{\pi}\right) = 225^\circ$

3.

θ	$-\pi$	$-\frac{2\pi}{3}$	0	$\frac{\pi}{2}$	$\frac{3\pi}{4}$
$\sin\theta$	0	$-\frac{\sqrt{3}}{2}$	0	1	$\frac{1}{\sqrt{2}}$
$\cos\theta$	-1	$-\frac{1}{2}$	1	0	$-\frac{1}{\sqrt{2}}$
$\tan\theta$	0	$\sqrt{3}$	0	und.	-1
$\cot\theta$	und.	$\frac{1}{\sqrt{3}}$	und.	0	-1
$\sec\theta$	-1	-2	1	und.	$-\sqrt{2}$
$\csc\theta$	und.	$-\frac{2}{\sqrt{3}}$	und.	1	$\sqrt{2}$

4.

θ	$-\frac{3\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{5\pi}{6}$
$\sin\theta$	1	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
$\cos\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$
$\tan\theta$	und.	$-\sqrt{3}$	$-\frac{1}{\sqrt{3}}$	1	$-\frac{1}{\sqrt{3}}$
$\cot\theta$	0	$-\frac{1}{\sqrt{3}}$	$-\sqrt{3}$	1	$-\sqrt{3}$
$\sec\theta$	und.	2	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	$-\frac{2}{\sqrt{3}}$
$\csc\theta$	1	$-\frac{2}{\sqrt{3}}$	-2	$\sqrt{2}$	2

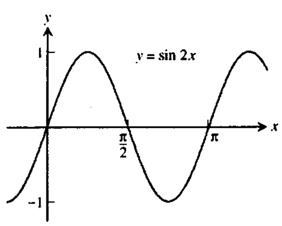
5. (a) $\cos x = -\frac{4}{5}$, $\tan x = -\frac{3}{4}$

   (b) $\sin x = -\frac{2\sqrt{2}}{3}$, $\tan x = -2\sqrt{2}$

6. (a) $\sin x = -\frac{1}{\sqrt{5}}$, $\cos x = -\frac{2}{\sqrt{5}}$

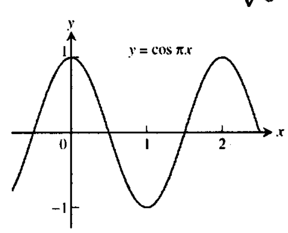
   (b) $\cos x = -\frac{\sqrt{3}}{2}$, $\tan x = \frac{1}{\sqrt{3}}$

7. (a)



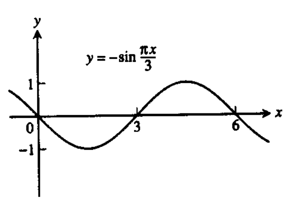
period $= \pi$

   (b)



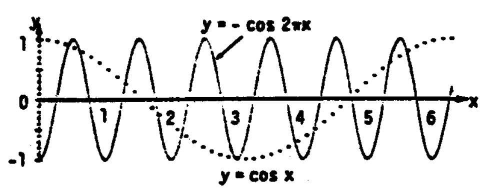
period $= 2$

8. (a)



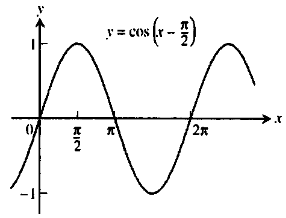
period $= 6$

   (b)



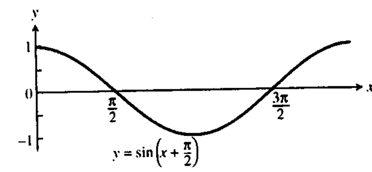
period $= 1$

9. (a)



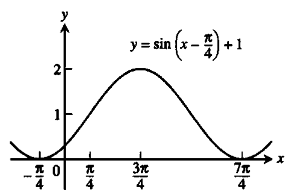
period = 2π

(b)



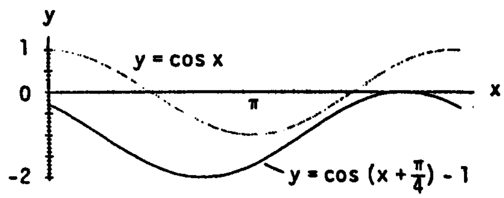
period = 2π

10. (a)



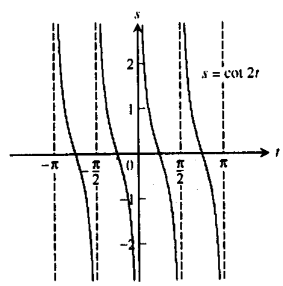
period = 2π

(b)

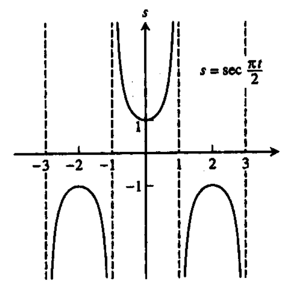


period = 2π

11. period = $\frac{\pi}{2}$, symmetric about the origin



12. period = 4, symmetric about the y-axis



13. (a) $\cos(\pi + x) = \cos\pi\cos x - \sin\pi\sin x = (-1)(\cos x) - (0)(\sin x) = -\cos x$
(b) $\sin(2\pi - x) = \sin 2\pi\cos(-x) + \cos(2\pi)\sin(-x) = (0)(\cos(-x)) + (1)(\sin(-x)) = -\sin x$

14. (a) $\sin\left(\frac{3\pi}{2} - x\right) = \sin\left(\frac{3\pi}{2}\right)\cos(-x) + \cos\left(\frac{3\pi}{2}\right)\sin(-x) = (-1)(\cos x) + (0)(\sin(-x)) = -\cos x$
(b) $\cos\left(\frac{3\pi}{2} + x\right) = \cos\left(\frac{3\pi}{2}\right)\cos x - \sin\left(\frac{3\pi}{2}\right)\sin x = (0)(\cos x) - (-1)(\sin x) = \sin x$

15. (a) $\cos\left(x - \frac{\pi}{2}\right) = \cos x\cos\left(-\frac{\pi}{2}\right) - \sin x\sin\left(-\frac{\pi}{2}\right) = (\cos x)(0) - (\sin x)(-1) = \sin x$

$$\cos(A - B) = \cos(A + (-B)) = \cos A\cos(-B) - \sin A\sin(-B) = \cos A\cos B - \sin A(-\sin B)$$
$$= \cos A\cos B + \sin A\sin B$$

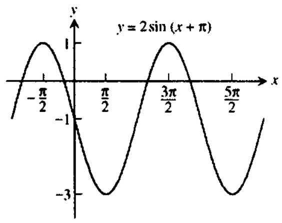
16. (a) $\sin\left(x + \frac{\pi}{2}\right) = \sin x\cos\left(\frac{\pi}{2}\right) + \cos x\sin\left(\frac{\pi}{2}\right) = (\sin x)(0) + (\cos x)(1) = \cos x$

(b) $\sin(A - B) = \sin(A + (-B)) = \sin A \cos(-B) + \cos A \sin(-B) = \sin A \cos B + \cos A(-\sin B)$
$= \sin A \cos B - \cos A \sin B$

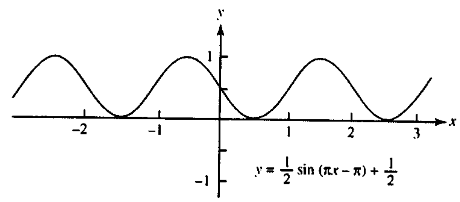
17. If $B = A$, $A - B = 0 \Rightarrow \cos(A - B) = \cos 0 = 1$. Also $\cos(A - B) = \cos(A - A) = \cos A \cos A + \sin A \sin A$ $= \cos^2 A + \sin^2 A$. Therefore, $\cos^2 A + \sin^2 A = 1$.

18. If $B = 2\pi$, then $\cos(A + 2\pi) = \cos A \cos 2\pi - \sin A \sin 2\pi = (\cos A)(1) - (\sin A)(0) = \cos A$ and $\sin(A + 2\pi) = \sin A \cos 2\pi + \cos A \sin 2\pi = (\sin A)(1) + (\cos A)(0) = \sin A$. The result agrees with the fact that the cosine and sine functions have period 2π.

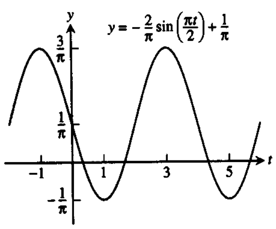
19. (a) $A = 2$, $B = 2\pi$, $C = -\pi$, $D = -1$



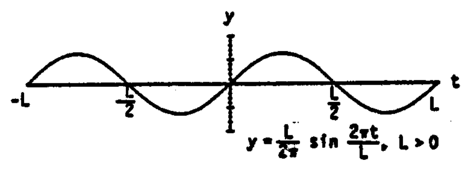
(b) $A = \frac{1}{2}$, $B = 2$, $C = 1$, $D = \frac{1}{2}$



20. (a) $A = -\frac{2}{\pi}$, $B = 4$, $C = 0$, $D = \frac{1}{\pi}$



(b) $A = \frac{L}{2\pi}$, $B = L$, $C = 0$, $D = 0$



21. (a) amplitude $= |A| = 37$
(b) period $= |B| = 365$
(c) right horizontal shift $= C = 101$
(d) upward vertical shift $= D = 25$

22. (a) It is highest when the value of the sine is 1 at $f(101) = 37 \sin(0) + 25 = 62°F$.
The lowest mean daily temp. is $37(-1) + 25 = -12°F$.
(b) The average of the highest and lowest mean daily temperatures $= \frac{62° + (-12)°}{2} = 25°F$.
The average of the sine function is its horizontal axis, $y = 25$.

23. (a) $\frac{\pi}{4}$ (b) $-\frac{\pi}{3}$ (c) $\frac{\pi}{6}$

24. (a) $-\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $-\frac{\pi}{3}$

25. (a) $\frac{\pi}{3}$ (b) $\frac{3\pi}{4}$ (c) $\frac{\pi}{6}$

26. (a) $\frac{3\pi}{4}$ (b) $\frac{\pi}{6}$ (c) $\frac{2\pi}{3}$

27. The angle α is the large angle between the wall and the right end of the blackboard minus the small angle between the left end of the blackboard and the wall $\Rightarrow \alpha = \cot^{-1}\left(\frac{x}{15}\right) - \cot^{-1}\left(\frac{x}{3}\right)$.

28. $65° + (90° - \beta) + (90° - \alpha) = 180° \Rightarrow \alpha = 65° - \beta = 65° - \tan^{-1}\left(\frac{21}{50}\right) \approx 65° - 22.78° \approx 42.22°$

29. According to the figure in the text, we have the following: By the law of cosines, $c^2 = a^2 + b^2 - 2ab \cos\theta$ $= 1^2 + 1^2 - 2\cos(A - B) = 2 - 2\cos(A - B)$. By distance formula, $c^2 = (\cos A - \cos B)^2 + (\sin A - \sin B)^2$

$= \cos^2 A - 2\cos A \cos B + \cos^2 B + \sin^2 A - 2\sin A \sin B + \sin^2 B = 2 - 2(\cos A \cos B + \sin A \sin B)$. Thus $c^2 = 2 - 2\cos(A - B) = 2 - 2(\cos A \cos B + \sin A \sin B) \Rightarrow \cos(A - B) = \cos A \cos B + \sin A \sin B$.

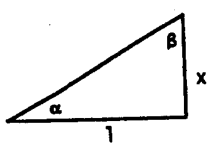
30. Consider the figure where $\theta = A + B$ is the sum of two angles. By the law of cosines, $c^2 = a^2 + b^2 - 2ab\cos\theta$
$= 1^2 + 1^2 - 2\cos(A + B) = 2 - 2\cos(A + B)$.
Also, by the distance formula,
$c^2 = (\cos A - \cos B)^2 + (\sin A + \sin B)^2$
$= \cos^2 A - 2\cos A \cos B + \cos^2 B + \sin^2 A + 2\sin A \sin B + \sin^2 B$
$= 2 - 2(\cos A \cos B - \sin A \sin B)$. Thus,
$2 - 2\cos(A + B) = 2 - 2(\cos A \cos B - \sin A \sin B)$
$\Rightarrow \cos(A + B) = \cos A \cos B - \sin A \sin B$.

31. Take each square as a unit square. From the diagram we have the following: the smallest angle α has a tangent of $1 \Rightarrow \alpha = \tan^{-1} 1$; the middle angle β has a tangent of $2 \Rightarrow \beta = \tan^{-1} 2$; and the largest angle γ has a tangent of $3 \Rightarrow \gamma = \tan^{-1} 3$. The sum of these three angles is $\pi \Rightarrow \alpha + \beta + \gamma = \pi$
$\Rightarrow \tan^{-1} 1 + \tan^{-1} 2 + \tan^{-1} 3 = \pi$.

32. (a) From the symmetry of the diagram, we see that $\pi - \sec^{-1} x$ is the vertical distance from the graph of $y = \sec^{-1} x$ to the line $y = \pi$ and this distance is the same as the height of $y = \sec^{-1} x$ above the x-axis at $-x$; i.e., $\pi - \sec^{-1} x = \sec^{-1}(-x)$.

(b) $\cos^{-1}(-x) = \pi - \cos^{-1} x$, where $-1 \le x \le 1 \Rightarrow \cos^{-1}\left(-\frac{1}{x}\right) = \pi - \cos^{-1}\left(\frac{1}{x}\right)$, where $x \ge 1$ or $x \le -1$
$\Rightarrow \sec^{-1}(-x) = \pi - \sec^{-1} x$

33. $\sin^{-1}(1) + \cos^{-1}(1) = \frac{\pi}{2} + 0 = \frac{\pi}{2}$; $\sin^{-1}(0) + \cos^{-1}(0) = 0 + \frac{\pi}{2} = \frac{\pi}{2}$; and $\sin^{-1}(-1) + \cos^{-1}(-1) = -\frac{\pi}{2} + \pi = \frac{\pi}{2}$.
If $x \in (-1, 0)$ and $x = -a$, then $\sin^{-1}(x) + \cos^{-1}(x) = \sin^{-1}(-a) + \cos^{-1}(-a) = -\sin^{-1} a + \left(\pi - \cos^{-1} a\right)$
$= \pi - \left(\sin^{-1} a + \cos^{-1} a\right) = \pi - \frac{\pi}{2} = \frac{\pi}{2}$ from Equations (7) and (9) in the text.

34. 

$\Rightarrow \tan\alpha = x$ and $\tan\beta = \frac{1}{x} \Rightarrow \frac{\pi}{2} = \alpha + \beta = \tan^{-1} x + \tan^{-1} \frac{1}{x}$.

35. From the figures in the text, we see that $\sin B = \frac{h}{c}$. If C is an acute angle, then $\sin C = \frac{h}{b}$. On the other hand, if C is obtuse (as in the figure on the right), then $\sin C = \sin(\pi - C) = \frac{h}{b}$. Thus, in either case, $h = b\sin C = c\sin B \Rightarrow ah = ab\sin C = ac\sin B$.

By the law of cosines, $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$ and $\cos B = \frac{a^2 + c^2 - b^2}{2ac}$. Moreover, since the sum of the interior angles of a triangle is π, we have $\sin A = \sin(\pi - (B + C)) = \sin(B + C) = \sin B \cos C + \cos B \sin C$
$= \left(\frac{h}{c}\right)\left[\frac{a^2 + b^2 - c^2}{2ab}\right] + \left[\frac{a^2 + c^2 - b^2}{2ac}\right]\left(\frac{h}{b}\right) = \left(\frac{h}{2abc}\right)\left(2a^2 + b^2 - c^2 + c^2 - b^2\right) = \frac{ah}{bc} \Rightarrow ah = bc\sin A$.

Combining our results we have ah = ab sin C, ah = ac sin B, and ah = bc sin A. Dividing by abc gives $\frac{h}{bc} = \underbrace{\frac{\sin A}{a} = \frac{\sin C}{c} = \frac{\sin B}{b}}_{\text{law of sines}}$.

36. $\tan(A+B) = \frac{\sin(A+B)}{\cos(A+B)} = \frac{\sin A \cos B + \cos A \cos B}{\cos A \cos B - \sin A \sin B} = \frac{\frac{\sin A \cos B}{\cos A \cos B} + \frac{\cos A \sin B}{\cos A \cos B}}{\frac{\cos A \cos B}{\cos A \cos B} - \frac{\sin A \sin B}{\cos A \cos B}} = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

37. (a) $c^2 = a^2 + b^2 - 2ab \cos C = 2^2 + 3^2 - 2(2)(3)\cos(60°) = 4 + 9 - 12 \cos(60°) = 13 - 12\left(\frac{1}{2}\right) = 7$. Thus, $c = \sqrt{7} \approx 2.65$.

(b) $c^2 = a^2 + b^2 - 2ab \cos C = 2^2 + 3^2 - 2(2)(3)\cos(40°) = 13 - 12 \cos(40°)$. Thus, $c = \sqrt{13 - 12 \cos 40°} \approx 1.951$.

38. (a) By the law of sines, $\frac{\sin A}{2} = \frac{\sin B}{3} = \frac{\sqrt{3}/2}{c}$. By Exercise 55 we know that $c = \sqrt{7}$. Thus $\sin B = \frac{3\sqrt{3}}{2\sqrt{7}} \simeq 0.982$.

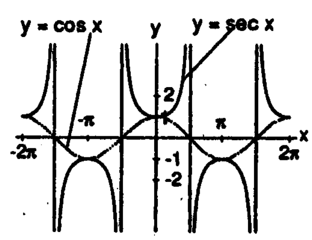
(a) From the figure at the right and the law of cosines, $b^2 = a^2 + 2^2 - 2(2a) \cos B = a^2 + 4 - 4a\left(\frac{1}{2}\right) = a^2 - 2a + 4$.

Applying the law of sines to the figure, $\frac{\sin A}{a} = \frac{\sin B}{b}$ $\Rightarrow \frac{\sqrt{2}/2}{a} = \frac{\sqrt{3}/2}{b} \Rightarrow b = \sqrt{\frac{3}{2}}a$. Thus, combining results, $a^2 - 2a + 4 = b^2 = \frac{3}{2}a^2 \Rightarrow 0 = \frac{1}{2}a^2 + 2a - 4$ $\Rightarrow 0 = a^2 + 4a - 8$. From the quadratic formula and the fact that $a > 0$, we have $a = \frac{-4 + \sqrt{4^2 - 4(1)(-8)}}{2} = \frac{4\sqrt{3} - 4}{2} \simeq 1.464$.

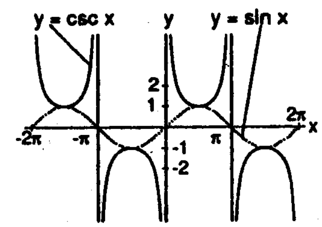
39. (a) The graphs of y = sin x and y = x nearly coincide when x is near the origin (when the calculator is in radians mode).

(b) In degree mode, when x is near zero degrees the sine of x is much closer to zero than x itself. The curves look like intersecting straight lines near the origin when the calculator is in degree mode.

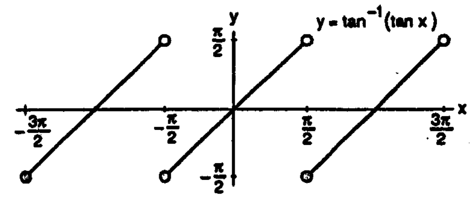
40. (a) Cos x and sec x are positive in QI and QIV and negative in QII and QIII. Sec x is undefined when cos x is 0. The range of sec x is $(-\infty, -1] \cup [1, \infty)$; the range of cos x is $[-1, 1]$.



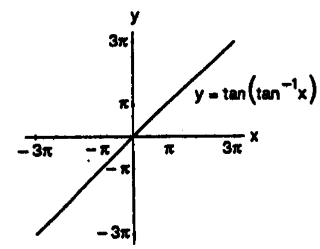
(b) Sin x and csc x are positive in QII and negative in QIII and QIV. Csc x is undefined when sin x is 0. The range of csc x is $(-\infty,-1]\cup[1,\infty)$; the range of sin x is $[-1,1]$.



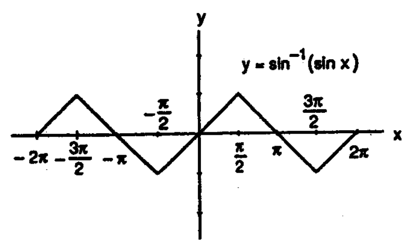
41. (a) Domain: all real numbers except those having the form $\frac{\pi}{2}+k\pi$ where k is an integer.
Range: $-\frac{\pi}{2}<y<\frac{\pi}{2}$



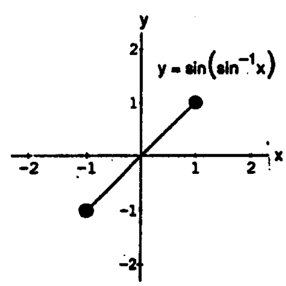
(b) Domain: $-\infty<x<\infty$; Range: $-\infty<y<\infty$
The graph of $y=\tan^{-1}(\tan x)$ is periodic, the graph of $y=\tan(\tan^{-1}x)=x$ for $-\infty\le x<\infty$.



42. (a) Domain: $-\infty<x<\infty$; Range: $-\frac{\pi}{2}\le y\le\frac{\pi}{2}$



(b) Domain: $-1\le x\le 1$; Range: $-1\le y\le 1$
The graph of $y=\sin^{-1}(\sin x)$ is periodic; the graph of $y=\sin(\sin^{-1}x)=x$ for $-1\le x\le 1$.



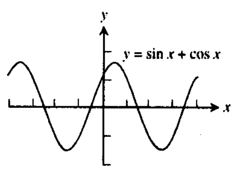
43. The angle $\tan^{-1}(2.5)\approx 1.190$ is the solution to this equation in the interval $-\frac{\pi}{2}<x<\frac{\pi}{2}$. Another solution in $0\le x<2\pi$ is $\tan^{-1}(2.5)+\pi\approx 4.332$. The solutions are $x\approx 1.190$ and $x\approx 4.332$.

44. The angle $\cos^{-1}(-0.7) \approx 2.346$ is the solution to this equation in the interval $0 \le x \le \pi$. Since the cosine function is even, the value $-\cos^{-1}(-0.7) \approx -2.346$ is also a solution, so any value of the form $\pm\cos^{-1}(-0.7) + 2k\pi$ is a solution, where k is an integer. In $2\pi \le x, 4\pi$ the solutions are $x = \cos^{-1}(-0.7) + 2\pi \approx 8.629$ and $x = -\cos^{-1}(-0.7) + 4\pi \approx 10.220$.

45. This equation is equivalent to $\cos x = -\frac{1}{3}$, so the solution in the interval $0 \le x \le \pi$ is $y = \cos^{-1}\left(-\frac{1}{3}\right) \approx 1.911$. Since the cosine function is even, the solutions in the interval $-\pi \le x < \pi$ are $x \approx -1.911$ and $x \approx 1.911$.

46. The solutions in the interval $0 \le x < 2\pi$ are $x = \frac{7\pi}{6}$ and $x = \frac{11\pi}{6}$. Since $y = \sin x$ has period 2π, the solutions are all of the form $x = \frac{7\pi}{6} + 2k\pi$ or $x = \frac{11\pi}{6} + 2k\pi$, where k is any integer.

47. (a)



The graph is a sine/cosine type graph, but it is shifted and has an amplitude greater than 1.

(b) Amplitude ≈ 1.414 (that is, $\sqrt{2}$)

Period $= 2\pi$

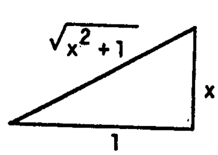
Horizontal shift ≈ -0.785 $\left(\text{that is, } -\frac{\pi}{4}\right)$ or 5.498 $\left(\text{that is, } \frac{7\pi}{4}\right)$ relative to sin x.

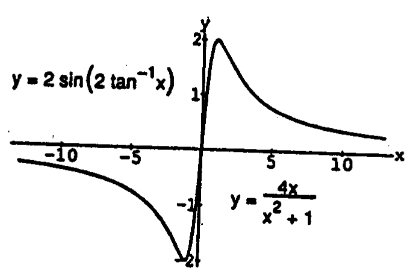
Vertical shift: 0

(c)
$$\sin\left(x + \frac{\pi}{4}\right) = (\sin x)\left(\cos\frac{\pi}{4}\right) + (\cos x)\left(\sin\frac{\pi}{4}\right)$$
$$= (\sin x)\left(\frac{1}{\sqrt{2}}\right) + (\cos x)\left(\frac{1}{\sqrt{2}}\right)$$
$$= \frac{1}{\sqrt{2}}(\sin x + \cos x)$$

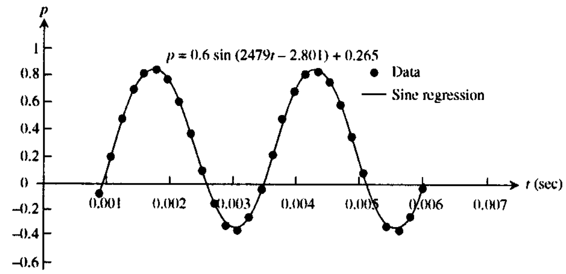
Therefore, $\sin x + \cos x = \sqrt{2}\,\sin\left(x + \frac{\pi}{4}\right)$

48. The graphs are identical for $y = 2\sin(2\tan^{-1}x)$
$$= 4[\sin(\tan^{-1}x)][\cos(\tan^{-1}x)] = 4\left(\frac{x}{\sqrt{x^2+1}}\right)\left(\frac{1}{\sqrt{x^2+1}}\right)$$
$$= \frac{4x}{x^2+1} \text{ from the triangle}$$





49. (a) The sinusoidal regression on the TI-92 Plus calculator gives $p = 0.599 \sin(2479t - 2.801) + 0.265$



(b) The period is approximately $\frac{2\pi}{2479}$ seconds, so the frequency is approximately $\frac{2479}{2\pi} \approx 395$ Hz

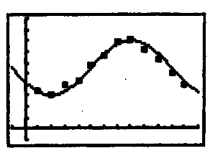
50. (a) $b = \frac{2\pi}{12} = \frac{\pi}{6}$

(b) It's half of the difference, so $a = \frac{80 - 30}{2} = 25$.

(c) $k = \frac{80 + 30}{2} = 55$

(d) The function should have its minimum at $t = 2$ (when the temperature is 30°F) and its maximum at $t = 8$ (when the temperature is 80°F). The value of h is $\frac{2 + 8}{2} = 5$. Equation: $y = 25 \sin\left[\frac{\pi}{6}(x - 5)\right] + 55$

(e)

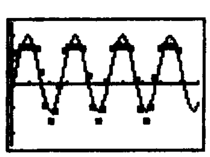


[−1, 13] by [−10, 100]

51. (a) Using a graphing calculator with the sinusoidal regression feature, the equation is $y = 3.0014 \sin(0.9996x + 2.0012) + 2.9999$.

(b) $y = 3 \sin(x + 2) + 3$

52. (a) Using a graphing calculator with the sinusoidal regression feature, the equation is $y = 1.543 \sin(2468.635x - 0.494) + 0.438$.

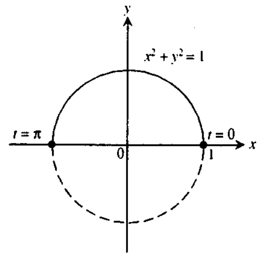


[0, 0.01] by [−2.5, 2.5]

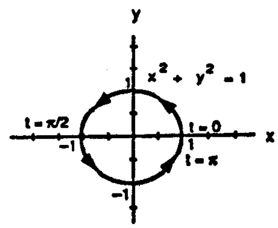
(b) The frequency is 2468.635 radians per second, which is equivalent to $\frac{2468.635}{2\pi} \approx 392.9$ cycles per second (Hz). The note is a "G."

P.6 PARAMETRIC EQUATIONS

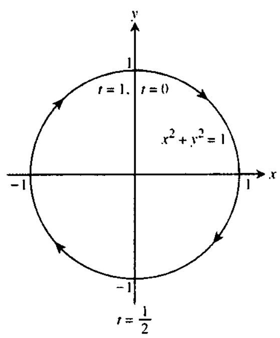
1. $x = \cos t,\ y = \sin t,\ 0 \le t \le \pi$
$\Rightarrow \cos^2 t + \sin^2 t = 1 \Rightarrow x^2 + y^2 = 1$



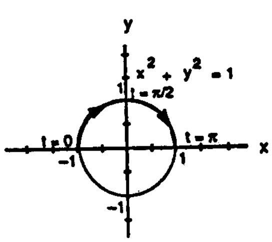
2. $x = \cos 2t,\ y = \sin 2t,\ 0 \le t \le \pi$
$\Rightarrow \cos^2 2t + \sin^2 2t = 1 \Rightarrow x^2 + y^2 = 1$



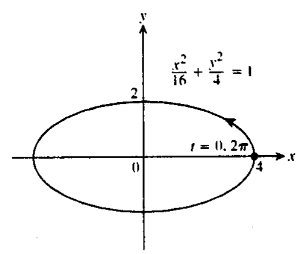
3. $x = \sin(2\pi t),\ y = \cos(2\pi t),\ 0 \le t \le 1$
$\sin^2(2\pi t) + \cos^2(2\pi t) = 1 \Rightarrow x^2 + y^2 = 1$



4. $x = \cos(\pi - t),\ y = \sin(\pi - t),\ 0 \le t \le \pi$
$\Rightarrow \cos^2(\pi - t) + \sin^2(\pi - t) = 1$
$\Rightarrow x^2 + y^2 = 1$

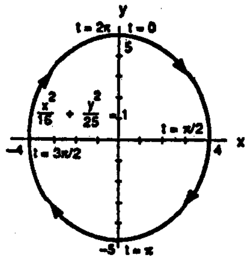


5. $x = 4 \cos t,\ y = 2 \sin t,\ 0 \le t \le 2\pi$
$\Rightarrow \frac{16 \cos^2 t}{16} + \frac{4 \sin^2 t}{4} = 1 \Rightarrow \frac{x^2}{16} + \frac{y^2}{4} = 1$

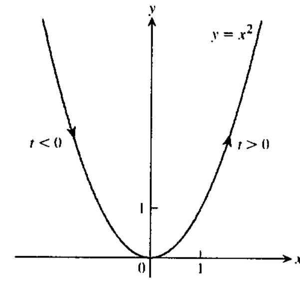


6. $x = 4 \sin t,\ y = 5 \cos t,\ 0 \le t \le 2\pi$

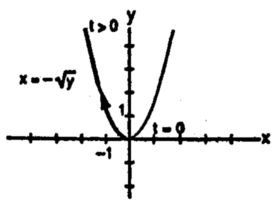
$\Rightarrow \frac{16 \sin^2 t}{16} + \frac{25 \cos^2 t}{25} = 1 \Rightarrow \frac{x^2}{16} + \frac{y^2}{25} = 1$



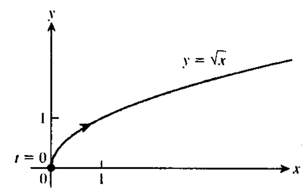
7. $x = 3t,\ y = 9t^2,\ -\infty < t < \infty \Rightarrow y = x^2$



8. $x = -\sqrt{t},\ y = t,\ t \ge 0 \Rightarrow x = -\sqrt{y}$
or $y = x^2,\ x \le 0$

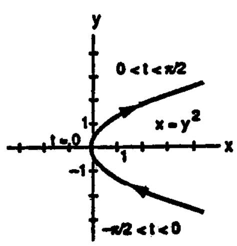


9. $x = t,\ y = \sqrt{t},\ t \ge 0 \Rightarrow y = \sqrt{x}$



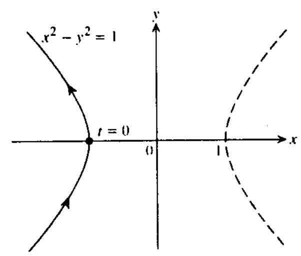
10. $x = \sec^2 t - 1,\ y = \tan t,\ -\frac{\pi}{2} < t < \frac{\pi}{2}$

$\Rightarrow \sec^2 t - 1 = \tan^2 t \Rightarrow x = y^2$



11. $x = -\sec t,\ y = \tan t,\ -\frac{\pi}{2} < t < \frac{\pi}{2}$

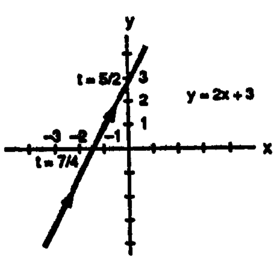
$\Rightarrow \sec^2 t - \tan^2 t = 1 \Rightarrow x^2 - y^2 = 1$



12. $x = 2t - 5,\ y = 4t - 7,\ -\infty < t < \infty$

$\Rightarrow x + 5 = 2t \Rightarrow 2(x + 5) = 4t$

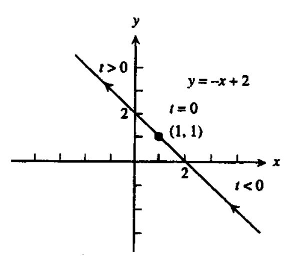
$\Rightarrow y = 2(x + 5) - 7 \Rightarrow y = 2x + 3$



13. $x = 1 - t,\ y = 1 + t,\ -\infty < t < \infty$

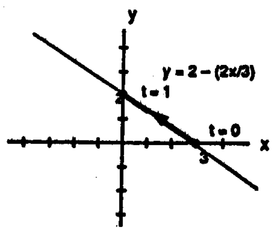
$\Rightarrow 1 - x = t \Rightarrow y = 1 + (1 - x)$

$\Rightarrow y = -x + 2$



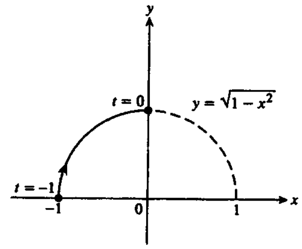
14. $x = 3 - 3t,\ y = 2t,\ 0 \le t \le 1 \Rightarrow \frac{y}{2} = t$

$\Rightarrow x = 3 - 3\left(\frac{y}{2}\right) \Rightarrow 2x = 6 - 3y \Rightarrow y = 2 - \frac{2}{3}x$



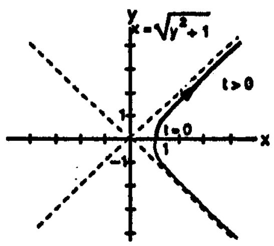
15. $x = t,\ y = \sqrt{1 - t^2},\ -1 \le t \le 0$

$\Rightarrow y = \sqrt{1 - x^2}$



16. $x = \sqrt{t + 1},\ y = \sqrt{t},\ t \ge 0$

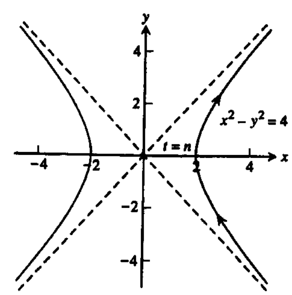
$\Rightarrow y^2 = t \Rightarrow x = \sqrt{y^2 + 1},\ y \ge 0$



17. $x = e^t + e^{-t},\ y = e^t - e^{-t},\ -\infty < t < \infty$

$\left(e^t + e^{-t}\right)^2 - \left(e^t - e^{-t}\right)^2 = \left(e^{2t} + 2 + e^{-2t}\right)$

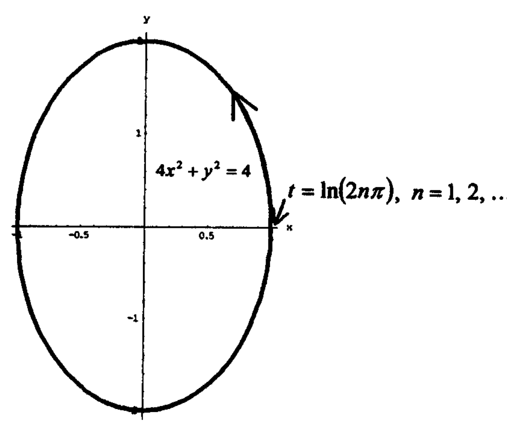
$- \left(e^{2t} - 2 + e^{-2t}\right) = 4 \Rightarrow x^2 - y^2 = 4$



18. $x = \cos\left(e^t\right),\ y = 2\sin\left(e^t\right),\ -\infty < t < \infty$

$\cos^2\left(e^t\right) + \sin^2\left(e^t\right) = 1 \Rightarrow x^2 + (y/2)^2 = 1$

$\Rightarrow 4x^2 + y^2 = 4$



19. (a) $x = a\cos t,\ y = -a\sin t,\ 0 \le t \le 2\pi$

(b) $x = a\cos t,\ y = a\sin t,\ 0 \le t \le 2\pi$

(c) $x = a\cos t,\ y = -a\sin t,\ 0 \le t \le 4\pi$

(d) $x = a\cos t,\ y = a\sin t,\ 0 \le t \le 4\pi$

20. (a) $x = a\sin t,\ y = b\cos t,\ \frac{\pi}{2} \le t \le \frac{5\pi}{2}$

(b) $x = a\cos t,\ y = b\sin t,\ 0 \le t \le 2\pi$

(c) $x = a\sin t,\ y = b\cos t,\ \frac{\pi}{2} \le t \le \frac{9\pi}{2}$

(d) $x = a\cos t,\ y = b\sin t,\ 0 \le t \le 4\pi$

21. Using $(-1, -3)$ we create the parametric equations $x = -1 + at$ and $y = -3 + bt$, representing a line which goes through $(-1, -3)$ at $t = 0$. We determine a and b so that the line goes through $(4, 1)$ when $t = 1$. Since $4 = -1 + a$, $a = 5$.

Since $1 = -3 + b$, $b = 4$.
Therefore, one possible parametrization is $x = -1 + 5t$, $y = -3 + 4t$, $0 \le t \le 1$.

22. Using $(-1, -3)$ we create the parametric equations $x = -1 + at$ and $y = 3 + bt$, representing a line which goes through $(-1, 3)$ at $t = 0$. We determine a and b so that the line goes through $(3, -2)$ $t = 1$.
Since $3 = -1 + a$, $a = 4$.
Since $-2 = 3 + b$, $b = -5$.
Therefore, one possible parametrization is $x = -1 + 4t$, $y = 3 - 5t$, $0 \le t \le 1$.

23. The lower half of the parabola is given by $x = y^2 + 1$ for $y \le 0$. Substituting t for y, we obtain one possible parametrization $x = t^2 + 1$, $y = t$, $t \le 0$.

24. The vertex of the parabola is at $(-1, -1)$, so the left half of the parabola is given by $y = x^2 + 2x$ for $x \le -1$. Substituting t for x, we obtain one possible parametrization: $x = t$, $y = t^2 + 2t$, $t \le -1$.

25. For simplicity, we assume that x and y are linear functions of t and that the point (x, y) starts at $(2, 3)$ for $t = 0$ and passes through $(-1, -1)$ at $t = 1$. Then $x = f(t)$, where $f(0) = 2$ and $f(1) = -1$.
Since slope $= \frac{\Delta x}{\Delta t} = \frac{-1-2}{1-0} = -3$, $x = f(t) = -3t + 2 = 2 - 3t$. Also, $y = g(t)$, where $g(t) = 3$ and $g(1) = -1$.
Since slope $= \frac{\Delta y}{\Delta t} = \frac{-1-3}{1-0} = -4$, $y = g(t) = -4t + 3 = 3 - 4t$.
One possible parametrization is: $x = 2 - 3t$, $y = 3 - 4t$, $t \ge 0$.

26. For simplicity, we assume that x and y are linear functions of t and that the point (x, y) starts at $(-1, 2)$ for $t = 0$ and passes through $(0, 0)$ at $t = 1$. Then $x = f(t)$, where $f(0) = -1$ and $f(1) = 0$.
Since slope $= \frac{\Delta x}{\Delta t} = \frac{0-(-1)}{1-0} = 1$, $x = f(t) = 1t + (-1) = -1 + t$. Also, $y = g(t)$, where $g(0) = 2$ and $g(1) = 0$.
Since slope $= \frac{\Delta y}{\Delta t} = \frac{0-2}{1-0} = -2$, $y = g(t) = -2t + 2 = 2 - 2t$.
One possible parametrization is: $x = -1 + t$, $y = 2 - 2t$, $t \ge 0$.

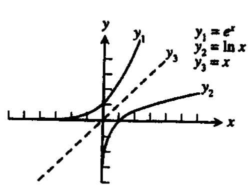
27. Graph (c). Window: $[-4, 4]$ by $[-3, 3]$, $0 \le t \le 2\pi$

28. Graph (a). Window: $[-2, 2]$ by $[-2, 2]$, $0 \le t \le 2\pi$

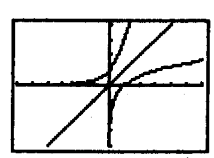
29. Graph (d). Window: $[-10, 10]$ by $[-10, 10]$, $0 \le t \le 2\pi$

30. Graph (b). Window: $[-15, 15]$ by $[-15, 15]$, $0 \le t \le 2\pi$

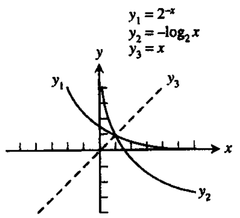
31. Graph of f: $x_1 = t$, $y_1 = e^t$
Graph of f^{-1}: $x_2 = e^t$, $y_2 = t$
Graph of $y = x$: $x_3 = t$, $y_3 = t$



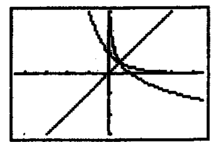
32. Graph of f: $x_1 = t$, $y_1 = 3^t$
Graph of f^{-1}: $x_2 = 3^t$, $y_2 = t$
Graph of $y = x$: $x_3 = t$, $y_3 = t$


$[-6, 6]$ by $[-4, 4]$

33. Graph of f: $x_1 = t$, $y_1 = 2^{-t}$
Graph of f^{-1}: $x_2 = 2^{-t}$, $y_2 = t$
Graph of $y = x$: $x_3 = t$, $y_3 = t$

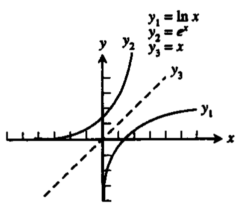


34. Graph of f: $x_1 = t$, $y_1 = 3^{-t}$
Graph of f^{-1}: $x_2 = 3^{-t}$, $y_2 = t$
Graph of $y = x$: $x_3 = t$, $y_3 = t$

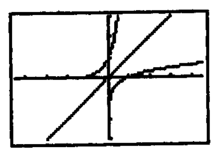


[−4.5, 4.5] by [−3, 3]

35. Graph of f: $x_1 = t$, $y_1 = \ln t$
Graph of f^{-1}: $x_2 = \ln t$, $y_2 = t$
Graph of $y = x$: $x_3 = t$, $y_3 = t$

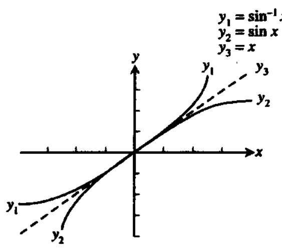


36. Graph of f: $x_1 = t$, $y_1 = \log t$
Graph of f^{-1}: $x_2 = \log t$, $y_2 = t$
Graph of $y = x$: $x_3 = t$, $y_3 = t$

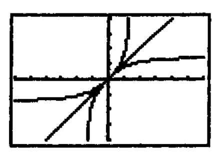


[−4.5, 4.5] by [−3, 3]

37. Graph of f: $x_1 = t$, $y_1 = \sin^{-1} t$
Graph of f^{-1}: $x_2 = \sin^{-1} t$, $y_2 = t$
Graph of $y = x$: $x_3 = t$, $y_3 = t$



38. Graph of f: $x_1 = t$, $y_1 = \tan^{-1} t$
Graph of f^{-1}: $x_2 = \tan^{-1} t$, $y_2 = t$
Graph of $y = x$: $x_3 = t$, $y_3 = t$



[−6, 6] by [−4, 4]

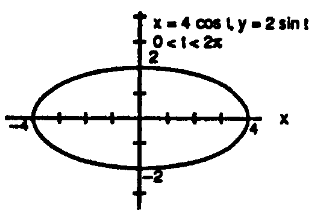
39. The graph is in Quadrant I when $0 < y < 2$, which corresponds to $1 < t < 3$. To confirm, note that $x(1) = 2$ and $x(3) = 0$.

40. The graph is in Quadrant II when $2 < y \le 4$, which corresponds to $3 < t \le 5$. To confirm, note that $x(3) = 0$ and $x(5) = -2$.

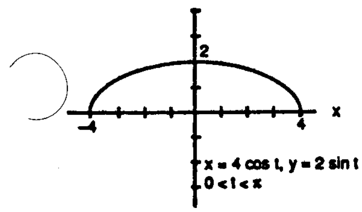
41. The graph is in Quadrant III when $-6 \le y < -4$, which corresponds to $-5 \le t < -3$. To confirm, note that $x(-5) = -2$ and $x(-3) = 0$.

42. The graph is in Quadrant IV when $-4 < y < 0$, which corresponds to $-3 < t < 1$. To confirm, note that $x(-3) = 0$ and $x(1) = 2$.

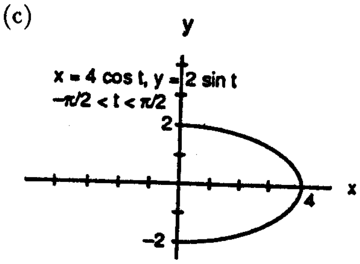
43. (a)



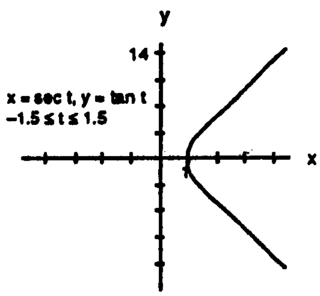
(b)



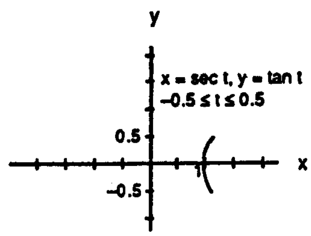
(c)



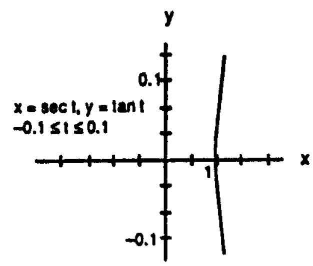
44. (a)



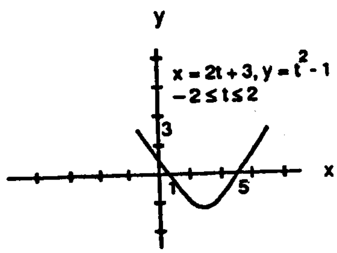
(b)



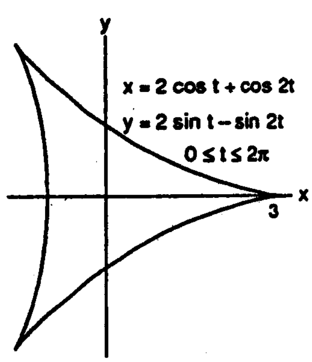
(c)



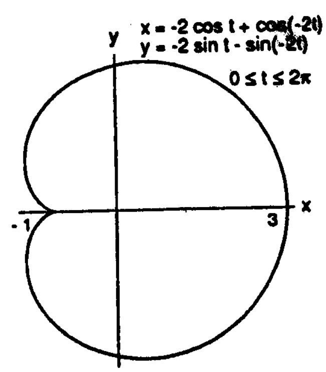
45.



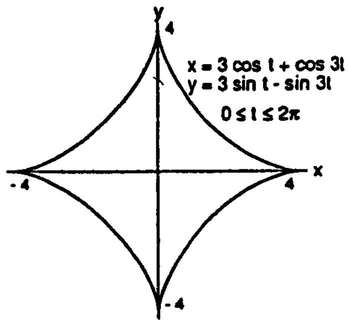
46. (a)



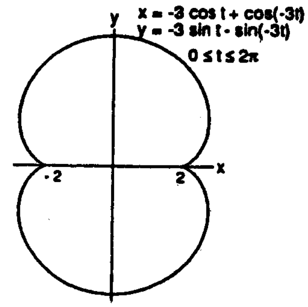
(b)



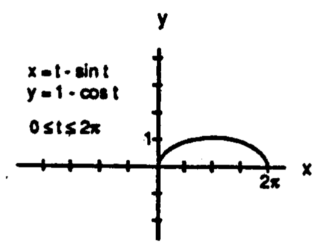
47. (a)



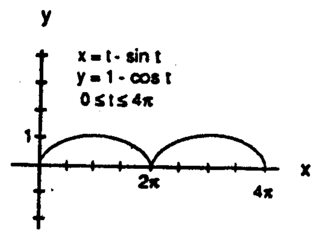
(b)



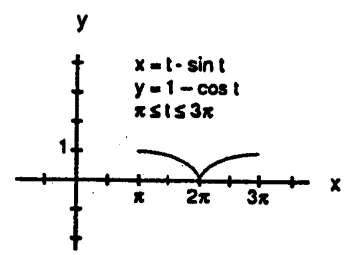
48. (a)



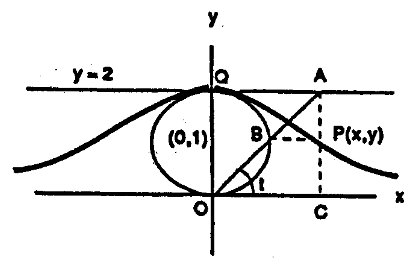
(b)



(c)



49. Extend the vertical line through A to the x-axis and let C be the point of intersection. Then OC = AQ = x and $\tan t = \frac{2}{OC} = \frac{2}{x} \Rightarrow x = \frac{2}{\tan t} = 2 \cot t$; $\sin t = \frac{2}{OA}$ $\Rightarrow OA = \frac{2}{\sin t}$; and $(AB)(OA) = (AQ)^2 \Rightarrow AB\left(\frac{2}{\sin t}\right) = x^2$ $\Rightarrow AB\left(\frac{2}{\sin t}\right) = \left(\frac{2}{\tan t}\right)^2 \Rightarrow AB = \frac{2 \sin t}{\tan^2 t}$. Next $y = 2 - AB \sin t \Rightarrow y = 2 - \left(\frac{2 \sin t}{\tan^2 t}\right) \sin t =$



$2 - \frac{2 \sin^2 t}{\tan^2 t} = 2 - 2\cos^2 t = 2 \sin^2 t$. Therefore let $x = 2 \cot t$ and $y = 2 \sin^2 t$, $0 < t < \pi$.

50. (a) $x = x_0 + (x_1 - x_0)t$ and $y = y_0 + (y_1 - y_0)t \Rightarrow t = \frac{x - x_0}{x_1 - x_0} \Rightarrow y = y_0 + (y_1 - y_0)\left(\frac{x - x_0}{x_1 - x_0}\right)$

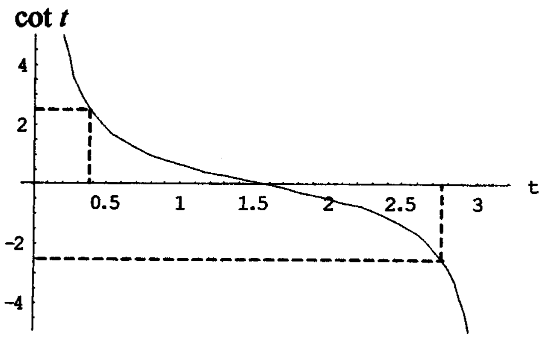
$\Rightarrow y - y_0 = \left(\frac{y_1 - y_0}{x_1 - x_0}\right)(x - x_0)$ which is an equation of the line through the points (x_0, y_0) and (x_1, y_1)

(b) Let $x_0 = y_0 = 0$ in (a) $\Rightarrow x = x_1 t$, $y = y_1 t$ (the answer is not unique)

(c) Let $(x_0, y_0) = (-1, 0)$ and $(x_1, y_1) = (0, 1)$ or let $(x_0, y_0) = (0, 1)$ and $(x_1, y_1) = (-1, 0)$ in part (a) $\Rightarrow x = -1 + t$, $y = t$ or $x = -t$, $y = 1 - t$ (the answer is not unique)

51. (a) $-5 \le x \le 5 \Rightarrow -5 \le 2 \cot t \le 5 \Rightarrow -\frac{5}{2} \le \cot t \le \frac{5}{2}$

The graph of cot t shows where to look for the limits on t.



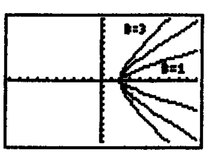
$\tan^{-1}\left(\frac{2}{5}\right) \leq t \leq \pi + \tan^{-1}\left(-\frac{2}{5}\right) \Rightarrow 0.381 \leq t \leq 2.761$

The curve is traced from right to left and extends infinitely in both directions from the origin.

(b) For $-\frac{\pi}{2} < t < \frac{\pi}{2}$, the curve is the same as that which is given. It first traces from the vertex at $(0, 2)$ to the left extreme point in the window, and then from the right extreme point in the window to the vertex point. For $0 < t < \frac{\pi}{2}$, only the right half of the curve appears, and it traces from the right extreme of the window to the vertex at $(0, 2)$ and terminates there. For $\frac{\pi}{2} < t < \pi$, only the left half of the curve appears, and it traces from the vertex to the left extreme of the window.

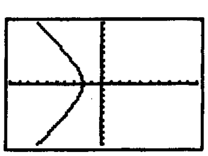
(c) For $x = -2 \cot t$, the curve traces from left to right rather than from right to left. For $x = 2 \cot(\pi - t)$, the curve traces from right to left, as it does with the original parametrization.

52. (a) The resulting graph appears to be the right half of a hyperbola in the first and fourth quadrants. The parameter a determines the x-intercept. The parameter b determines the shape of the hyperbola. If b is smaller, the graph has less steep slopes and appears "sharper." If b is larger, the slopes are steeper and the graph appears more "blunt." The graphs for a = 2 and b = 1, 2, and 3 are shown.



[−10, 10] by [−10, 10]

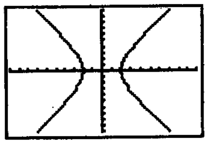
(b)



[−10, 10] by [−10, 10]

This appears to be the left half of the same hyperbola.

(c)



[−10, 10] by [−10, 10]

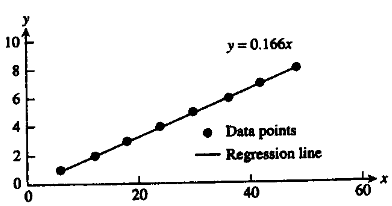
One must be careful because both sec t and tan t are discontinuous at these points. This might cause the grapher to include extraneous lines (the asymptotes of the hyperbola) in its graph. The extraneous lines can be avoided by using the grapher's dot mode instead of connected mode.

(d) Note that $\sec^2 t - \tan^2 t = 1$ by a standard trigonometric identity. Substituting $\frac{x}{a}$ for sec t and $\frac{y}{b}$ for tan t gives $\left(\frac{x}{a}\right)^2 - \left(\frac{y}{b}\right)^2 = 1$.

(e) This changes the orientation of the hyperbola. In this case, b determines the y-intercept of the hyperbola, and a determines the shape. The parameter interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ gives the upper half of the hyperbola. The parameter interval $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$ gives the lower half. The same values of t cause discontinuities and may add extraneous lines to the graph. Substituting $\frac{y}{b}$ for sec t and $\frac{x}{a}$ for tan t in the identity $\sec^2 t - \tan^2 t = 1$ gives $\left(\frac{y}{b}\right)^2 - \left(\frac{x}{a}\right)^2 = 1$.

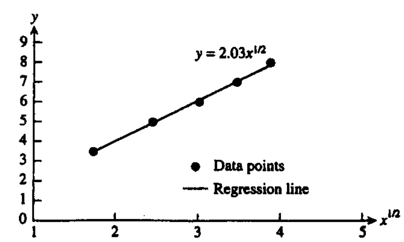
P.7 MODELING CHANGE

1. (a)



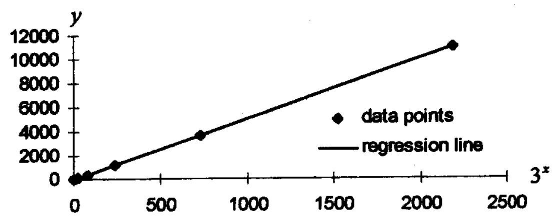
The graph supports the assumption that y is proportional to x. The constant of proportionality is estimated from the slope of the regression line, which is 0.166.

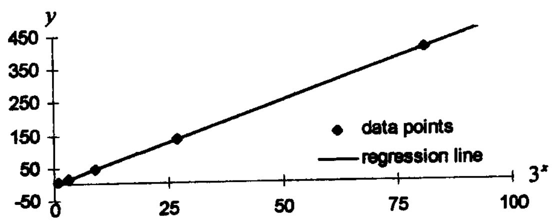
(b)



The graph supports the assumption that y is proportional to $x^{1/2}$. The constant of proportionality is estimated from the slope of the regression line, which is 2.03.

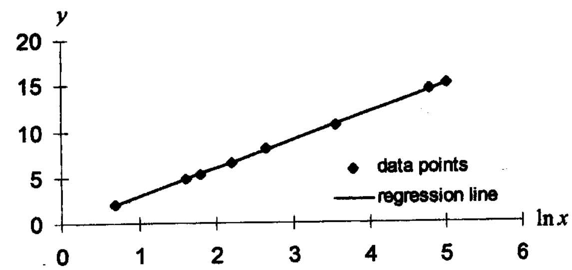
(c) Because of the wide range of values of the data, two graphs are needed to observe all of the points in relation to the regression line.





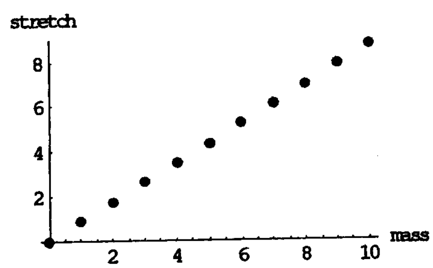
The graphs support the assumption that y is proportional to 3^x. The constant of proportionality is estimated from the slope of the regression line, which is 5.00.

(d)

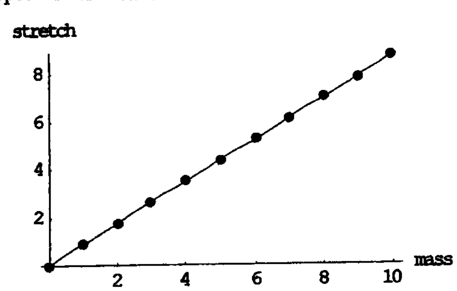


The graph supports the assumption that y is proportional to ln x. The constant of proportionality is estimated from the slope of the regression line, which is 2.99.

2. (a) Plot the data to see if there is a recognizable pattern.

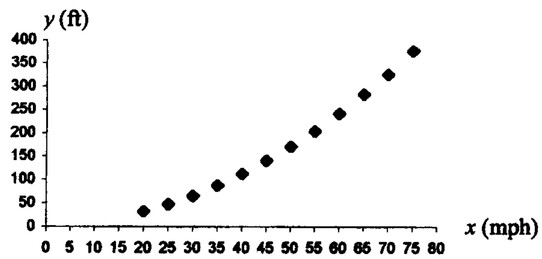


The data clearly suggests a linear relationship. The line of best fit, or the regression line, is s = 0.8742m where s is the stretch in the spring and m is the mass. Now we superimpose the regression line on the graph of the data.

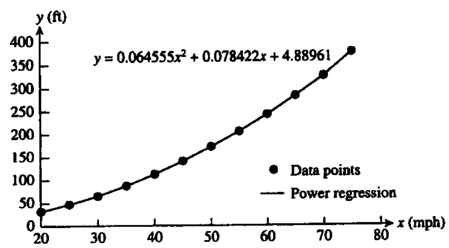


(b) The model fits the data very well.
(c) When m = 13, the model gives a stretch of s = 0.8742(13) = 11.365. Since this data point is outside the range of the data that the model is based upon, one should feel uncomfortable with this prediction of the stretch without further experimental verification.

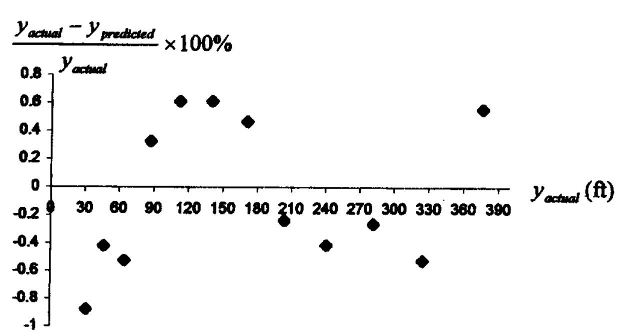
3. First, plot the braking distance versus the speed.



The shape of the graph suggests either a power function or an exponential function to describe the relationship. First, try to fit a quadratic function. Using quadratic regression on the TI-92 Plus calculator gives $y = 0.064555x^2 + 0.078422x + 4.88961$.



The quadratic regression fits the data well as seen by the following plot of the relative errors versus the actual stopping distance.

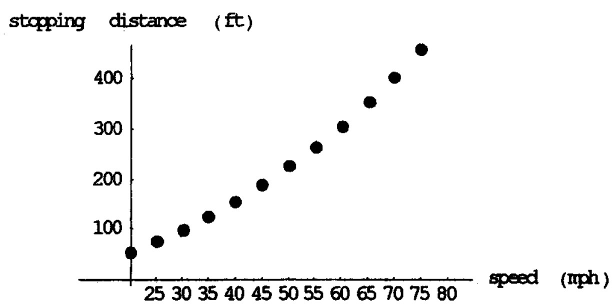


The largest relative error is less than 1%.

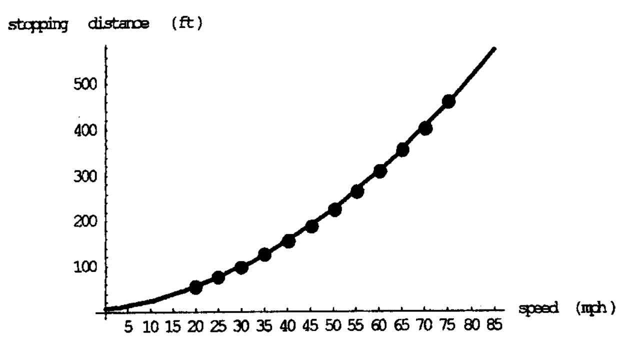
4. The following table gives the total stopping distance (reaction distance + braking distance) for automobile speeds ranging from 20 to 75 miles per hour.

speed	20	25	30	35	40	45	50	55	60	65	70	75
stopping distance	54	75	98	126	156	190	226	265	307	354	402	459

Plot the total stopping distance versus speed.



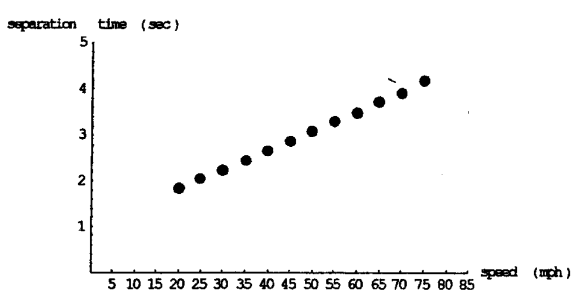
The graph suggests a possible quadratic relationship. Quadratic regression on the data gives $d = 0.0646v^2 + 1.181v + 5.040$ where d is the total stopping distance in feet and v is the travel speed in miles per hour. Now superimpose the quadratic regression on the graph of the data.



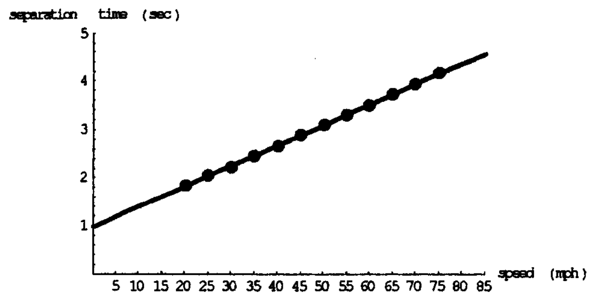
The quadratic regression fits the data very well. To test the 2-second "rule of thumb," calculate the time the vehicle will travel the distance d when it is traveling at speed v. (Don't forget to convert mph into ft/sec using 60 mph = 88 ft/sec.) The following table gives separation times versus travel speed.

v (mph)	20	25	30	35	40	45	50	55	60	65	70	75
t (sec)	1.84	2.05	2.23	2.45	2.66	2.88	3.08	3.29	3.49	3.71	3.92	4.17

Plot the data.

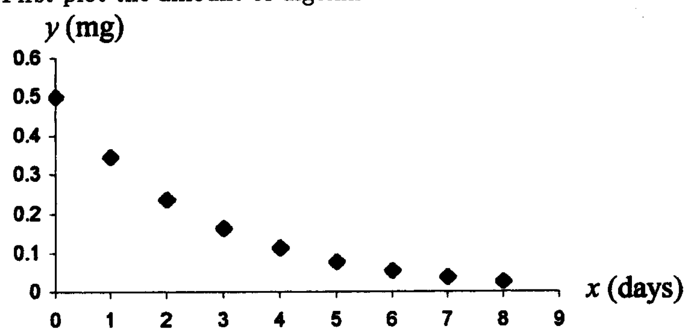


The graph suggests a linear relationship and a linear regression gives $t = 0.042v + 0.983$. Now superimpose the linear regression function on the graph of the data.

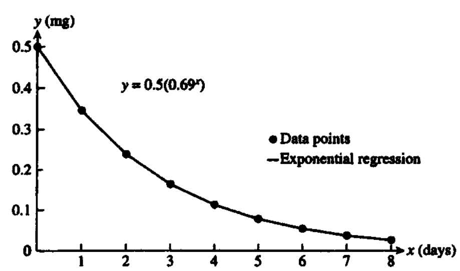


Based on the preceding analysis, a better rule of thumb would be to keep a minimum separation time of 2 seconds and add 1 sec for every 20 mph increment of speed above 20 mph. So, for example, if you are traveling at 40 mph your separation should be $2 + 1(1) = 3$ seconds, at 60 mph your separation should be $2 + 2(1) = 4$ seconds, at 80 mph it should be $2 + 3(1) = 5$ seconds, and so on.

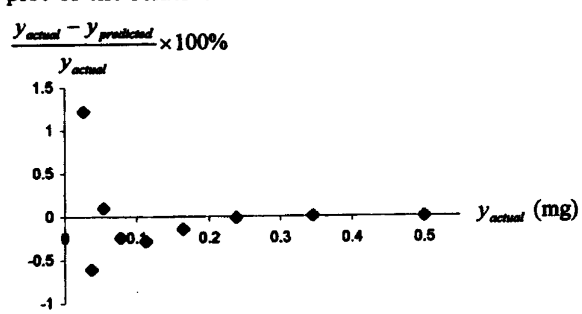
5. (a) First plot the amount of digoxin in the blood versus time.



The graph suggests that the amount decays exponentially with time. The exponential regression on the TI-92 Plus calculator gives $y = 0.5(0.69^x) = 0.5e^{-0.371x}$.

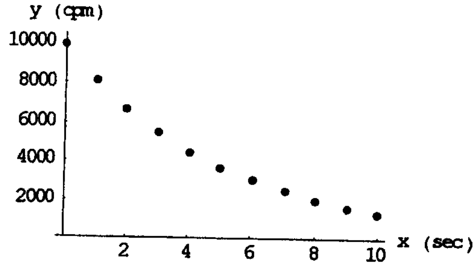


(b) The exponential function fits the data very well as demonstrated by the graph above and the following is a plot of the relative error versus the actual amount in the blood.

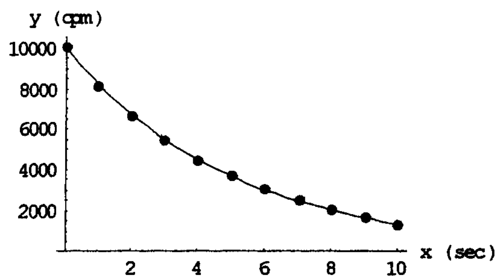


(c) $y(12) = 0.5e^{-0.371(12)} = 0.00583$, therefore, the model predicts that after 12 hours, the amount of digoxin in the blood will be less than 0.006 mg.

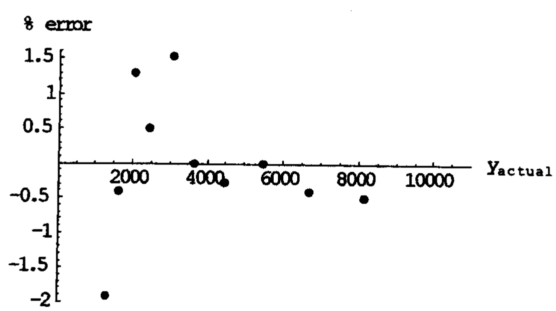
6. (a) Plot the data.



An exponential regression on the TI-92 Plus calculator gives $y = 10{,}037e^{-0.2005x}$. Superimpose the regression function on the graph of the data.



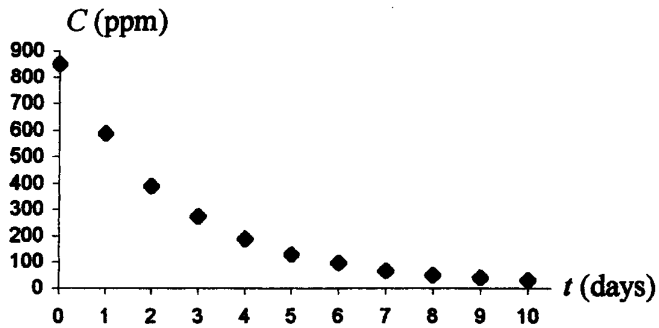
(b) The exponential function fits the data very well as indicated by the graph above. The following is a graph of the relative error, $\frac{y_{predicted} - y_{actual}}{y_{actual}} \times 100\%$, versus y_{actual}.



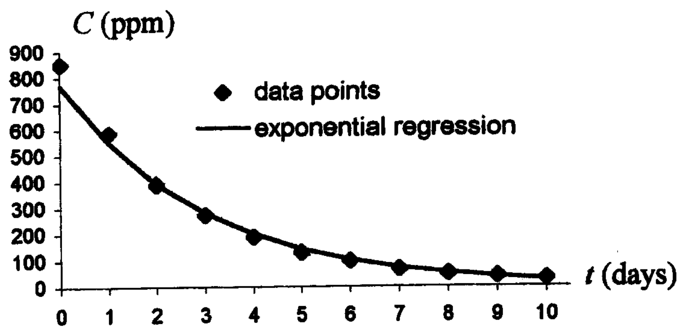
The largest relative error is less than 2% in magnitude.

(c) $500 = 10{,}037e^{-0.2005x} \Rightarrow -0.2005x = \ln\left(\frac{500}{10{,}037}\right) \Rightarrow x = 15.0$ minutes.

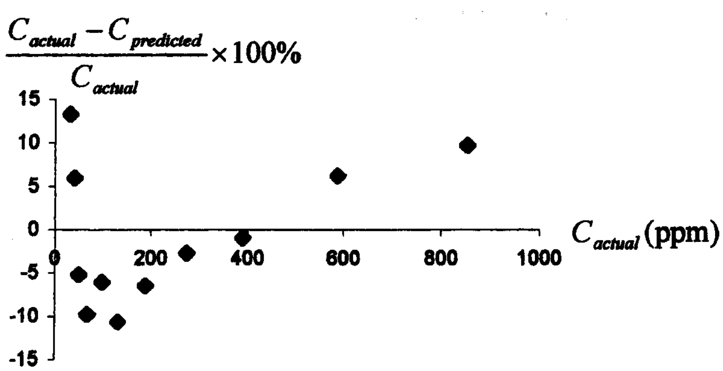
7. (a) First, plot a graph of the blood concentration versus time. Let t represent the elapsed time in days and C the blood concentration in parts per million.



The graph suggests that the amount decays exponentially with time. The exponential regression function on the TI-92 Plus calculator gives $C = 770(0.7146^t) = 770e^{-0.336t}$.



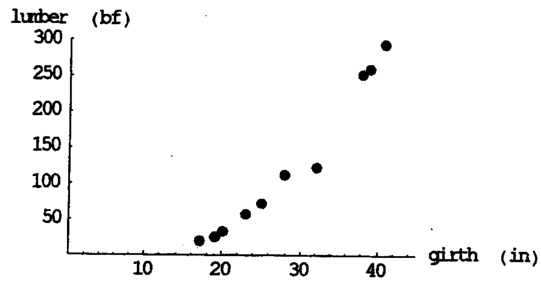
(b) The exponential function appears to capture a trend for this data. The following graph shows the relative errors in the model estimates.



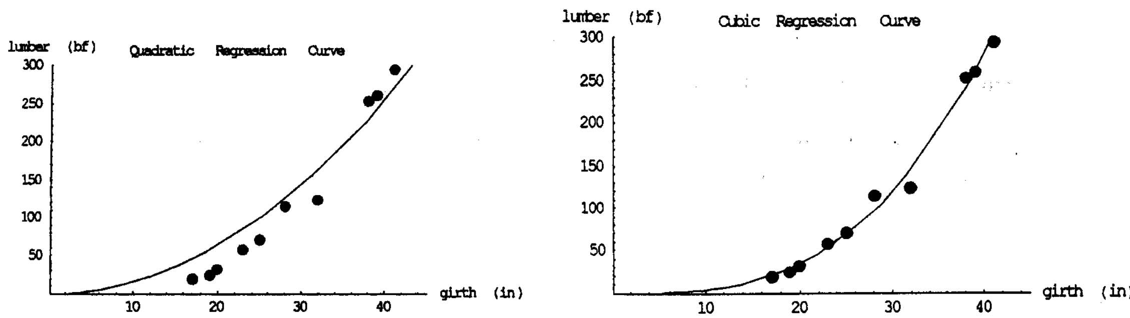
The relative errors in the predicted values are as large as 13.2% and the errors are large for small as well as large blood concentrations. The pattern of the residual errors does not suggest an obvious improvement of the model.

(c) $10 = 770e^{-0.336t} \Rightarrow t = -\frac{1}{0.336}\ln\left(\frac{10}{770}\right) = 12.93$ days. Therefore, the model predicts that the blood concentration will fall below 10 ppm after 12 days and 22 hours.

8. Plot the data.

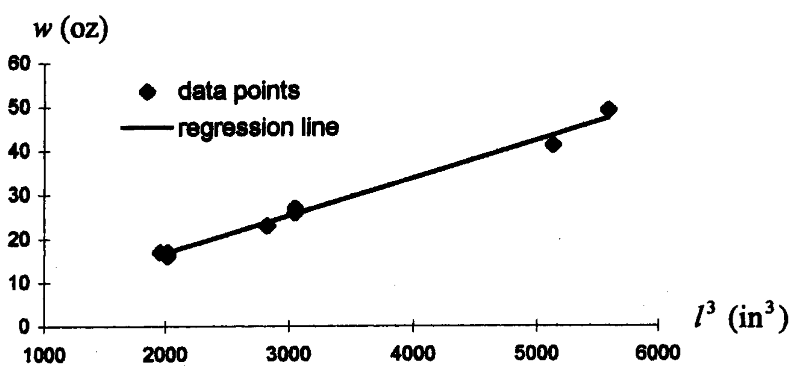


(a) and (b) The quadratic regression function is $y = 0.1579x^2$ where x represents the girth in inches and y the amount of usable lumber in board feet. The cubic regression function is $y = 0.00436x^3$. Superimpose the two regression functions on the graph of the data.

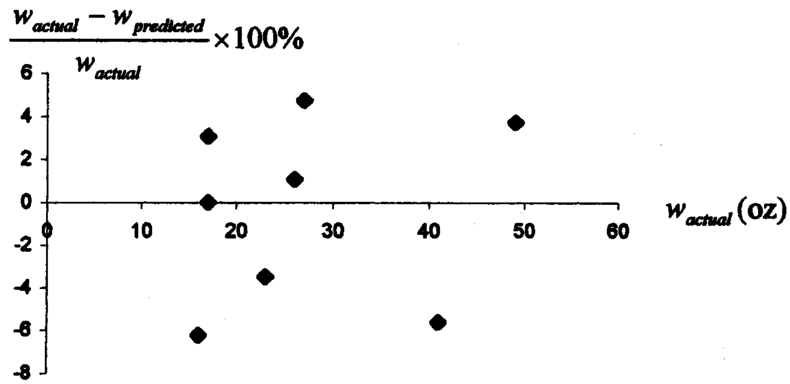


The graphs show that the cubic relationship provides the better model.
Explanation of the model: The unit of board feet is a measure of the volume and, if a tree is modeled as a right circular cone, its volume would be $y = \frac{1}{3}\pi r^2 h$. The girth is the circumference of the tree near the base so that $x = 2\pi r \Rightarrow r = \frac{x}{2\pi}$. If, in addition, we assume that as a tree grows the proportion $\frac{h}{r} = k$, a constant, then we have that $y = \frac{1}{3}\pi\left(\frac{x}{2\pi}\right)^2(kr) = \frac{1}{3}\pi\left(\frac{x}{2\pi}\right)^2\left(\frac{kx}{2\pi}\right) = \frac{k}{24\pi}x^3$, which shows that $y = 0.00436x^3$ is a rational model.

9.

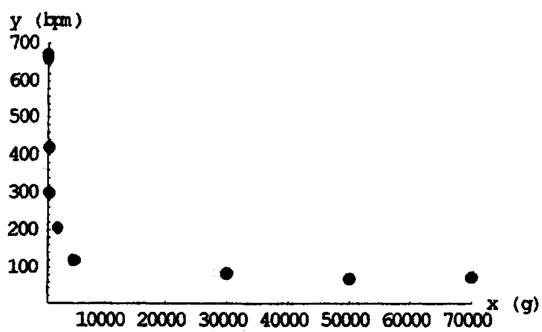


The slope of the regression line is 0.008435, so the model that estimates the weight as a function of L is $w = 0.008435L^3$. The model fits the data reasonably well as demonstrated by the following plot of the relative errors in the weight estimates by the model.

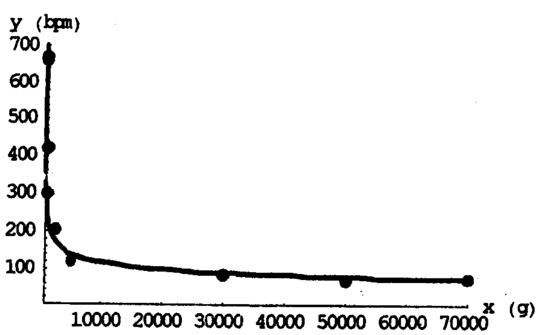


The relative error in the estimated values is always less than 7%.

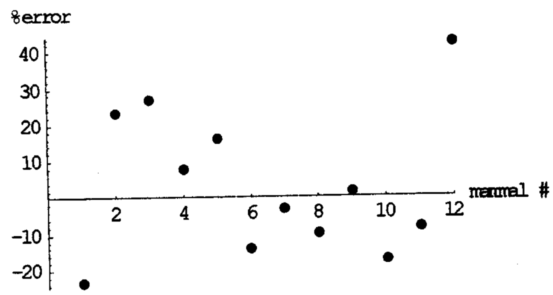
10. The following plot of the data does not include the ox and elephant. However, the data for these mammals are used in the analysis that follows.



There does appear to be a trend. After trying regressions with n = 1, 2, 3, 4, 5, the best fit was found with n = 4. The following graph superimposes the regression function $y = 1150x^{-1/4}$ on the data points.



To test the model, calculate the relative errors (i.e., $\frac{y_{predicted} - y_{actual}}{y_{actual}} \times 100\%$) for all of the mammals in the sample set. These are shown in the following graph.



The errors appear to be random and the largest relative errors are for the two larger animals (i.e., the ox and the elephant) with magnitudes of 92% and 43%, respectively. The model appears to capture a trend in the data, which could be useful in understanding the relationship between mammal size and heart rate; however, it probably would not be useful as a predictive tool.

11. Graph (c). For some drugs, the rate of elimination is proportional to the concentration of the drug in the blood-stream. Graph (c) matches this behavior because the graph falls faster at higher concentrations.

12. Graph (d) or graph (f). Often times, when we begin to learn a new subject, we master the basics quickly at first, but then as the subject becomes more intricate our proficiency increases more slowly. This learning behavior would be described by graph (d). Some subjects have high overhead in terms of learning basic skills and so our proficiency increases slowly at first but, as we acquire the basic skills, our proficiency increases more rapidly. Then, as we reach our intellectual capacity or as our interest wanes, our proficiency will increase more slowly. Graph (f) would match this learning pattern.

13. Graph (c). The rate of decay of radioactive Carbon-14 is proportional to the amount of Carbon-14 present in the artwork. Graph (c) matches this behavior because the graph falls faster at higher amounts.

14. (a) Graph (e). At first, the water velocity is high but as the tank drains the velocity will decrease. When the water level in the tank is high the discharge velocity will decrease slowly, but as time progresses and the water level drops, the discharge velocity will decrease more rapidly.
    (b) Graph (c). Assuming the tank is an upright circular cylinder, the rate at which the water level in the tank falls will be proportional to the rate at which the volume of water in the tank decreases. Also, the rate at which the volume decreases will be proportional to the discharge velocity. Therefore, when the discharge velocity is high at the start, the rate at which the volume decreases will be high and so will the rate of decrease in the water depth. As the discharge velocity decreases, the rate at which the water depth drops will also decrease. This behavior is depicted by graph (c).

15. (a) One possibility: If an item sells for $p and x is the number of items sold, then the revenue from sales will be $y = px$, and the graph of the revenue function looks like graph (a).
    (b) One possibility: If y is the number of deer in a very large game reserve with unlimited resources to support the deer and x represents the number of years elapsed, then the population would exhibit unconstrained growth over time. In this situation, the population can be modeled by an exponential growth function like $y = y_0 e^{kx}$, where y_0 is the initial deer population, k is a constant, and the growth of the function looks like graph (b).
    (c) One possibility: If y represents the selling price per unit that can be realized for a certain commodity, say grape jelly for example, and x represents the availability of the commodity, then the unit selling price for the commodity is often times inversely proportional to its availability. This relationship can be modeled with a function of the form $y = \frac{y_0}{(x+1)^\alpha}$, where y_0 is the unit selling price when no product is available, α is a positive constant, and the graph of the function looks like graph (c).

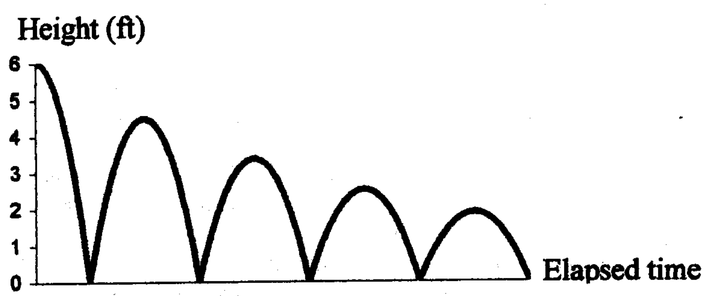
(d) One possibility: Let y represent the speed of your car and x represent the amount of time after you punch the accelerator. At first you will rapidly accelerate but, as the car picks up speed, the rate of acceleration (i.e., the rate at which the car speeds up) decreases. This can be modeled by a function like $y = y_{new} + (y_0 - y_{new})e^{-kx}$, where y_0 is the speed you were traveling when you stepped on it, y_{new} is the new speed you achieve when you are done accelerating, and k is a positive constant (determined in part by the size of your engine and how good your traction is). The graph of this function looks like graph(d).

(e) One possibility: Let y represent the amount you owe on your credit card and x represent the number of monthly payments you have made. At first the amount you owe decreases slowly because most of your payment goes toward paying the monthly interest charge. But, as the amount you owe decreases, the interest charge decreases and your payment makes a bigger difference toward reducing the debt. This can be modeled with a function like the one represented by graph (c).

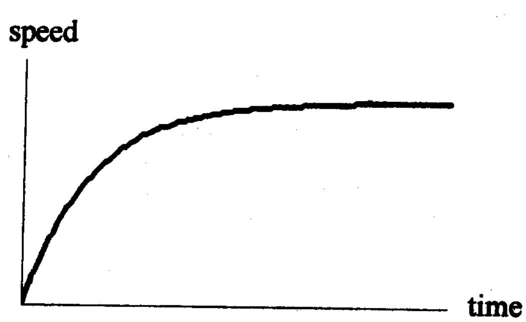
(f) One possibility: Let y represent the number of people in your school who have the flu and let x represent the number of days that have elapsed after the first person gets sick. At first the flu doesn't spread very quickly because there are only a few sick people to pass it on. But, as more people get sick the disease spreads more rapidly. The most volatile mixture is when half the people are sick, because then there are a lot of sick people to spread the disease and a lot of uninfected people who can still catch it. As time continues and more people get sick, there are fewer and fewer people available to catch the flu and the spread of the disease begins to slow down. This behavior can be modeled with a function like the one represented by graph (f).

16. The intensity of light will probable decrease linearly as the number of layers of plastic increases. If I_0 is the intensity with no layers of plastic, then the relationship would be $I = I_0 - kn$, where n is the number of layers and k is a constant.

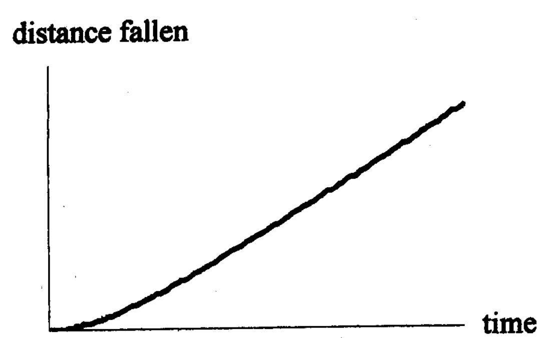
17.



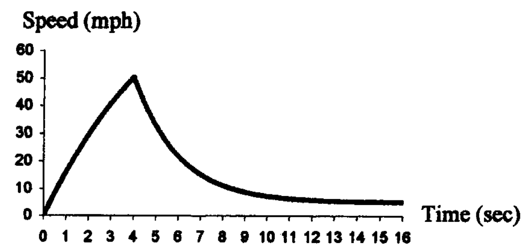
18. (a)



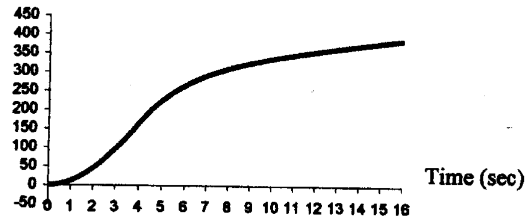
(b)



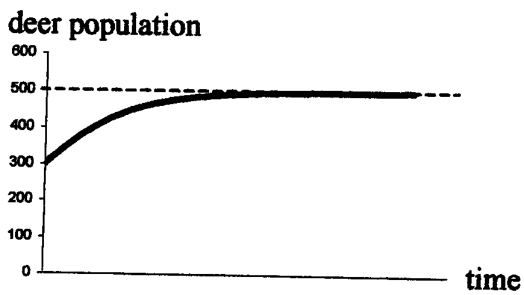
19. (a)



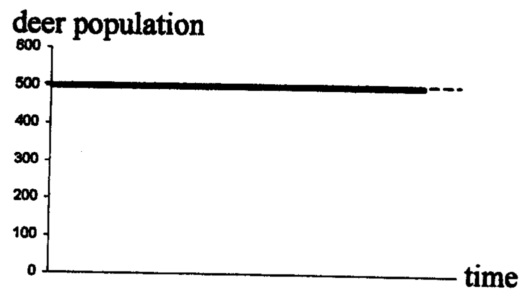
(b)



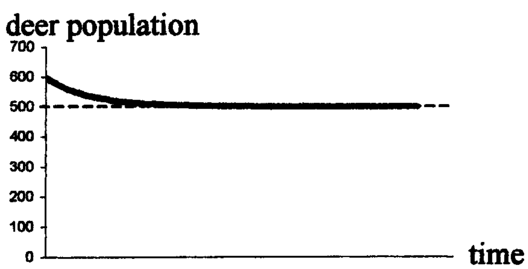
20. (a)



(b)



(c)



21. (a) The graph could represent the angle that a pendulum makes with the vertical as it swings back and forth. The variable y represents the angle and x represents time. Because of friction, the amplitude of the oscillation decays, as depicted by the graph. When y is positive, the pendulum is on one side of the vertical and when y is negative, the pendulum is on the other side.

(b) The graph could represent the angle the playground swing makes with the vertical as a child "pumps" on the swing to get it going. The variable y represents the angle and x represents time. Because the child puts mechanical energy into the system (swing + child), the amplitude of the oscillation grows with time, as depicted by the graph. When y is positive, the swing is on one side of the vertical and when y is negative, the swing is on the other side.

22. Answers will vary. An example follows.

(f) I would like to study the effect that the geometric configuration of a group of four light poles and fixtures would have on the illumination intensity on the ground. More specifically, if four light poles are arranged in a square, how is the light intensity on the ground at the center of the square (where the light intensity is assumed to be minimum) affected by the spacing of the poles? To determine the effect of the pole spacing, I will need to design an experiment to measure the intensity of light at the center of the square pattern as the pole spacing is varied. After collecting the data I would then try to find a mathematical model to fit the data. The parking lot designer could use this model to determine the maximum pole spacing, given the minimum required light intensity on the ground. In addition to the spacing of the poles, some other variables that affect the illumination are the type, size and brightness of the light sources, shadowing by other objects, and the height of the poles.

PRELIMINARY CHAPTER PRACTICE EXERCISES

1. $y = 3(x-1) + (-6)$
   $y = 3x - 9$

2. $y = -\frac{1}{2}(x+1) + 2$

   $y = -\frac{1}{2}x + \frac{3}{2}$

3. $x = 0$

4. $m = \frac{-2-6}{1-(-3)} = \frac{-8}{4} = -2$

   $y = -2(x+3) + 6$

   $y = -2x$

5. $y = 2$

6. $m = \frac{5-3}{-2-3} = \frac{2}{-5} = -\frac{2}{5}$

   $y = -\frac{2}{5}(x-3) + 3$

   $y = -\frac{2}{5}x + \frac{21}{5}$

7. $y = -3x + 3$

8. Since $2x - y = -2$ is equivalent to $y = 2x + 2$, the slope of the given line (and hence the slope of the desired line) is 2.
   $y = 2(x-3) + 1$
   $y = 2x - 5$

9. Since $4x + 3y = 12$ is equivalent to $y = -\frac{4}{3}x + 4$, the slope of the given line (and hence the slope of the desired line) is $-\frac{4}{3}$.

   $y = -\frac{4}{3}(x-4) - 12$

   $y = -\frac{4}{3}x - \frac{20}{3}$

10. Since $3x - 5y = 1$ is equivalent to $y = \frac{3}{5}x - \frac{1}{5}$, the slope of the given line is $\frac{3}{5}$ and the slope of the perpendicular line is $-\frac{5}{3}$.

$y = -\frac{5}{3}(x+2) - 3$

$y = -\frac{5}{3}x - \frac{19}{3}$

11. Since $\frac{1}{2}x + \frac{1}{3}y = 1$ is equivalent to $y = -\frac{3}{2}x + 3$, the slope of the given line is $-\frac{3}{2}$ and the slope of the perpendicular line is $\frac{2}{3}$.

$y = \frac{2}{3}(x+1) + 2$

$y = \frac{2}{3}x + \frac{8}{3}$

12. The line passes through $(0,-5)$ and $(3,0)$.

$m = \frac{0-(-5)}{3-0} = \frac{5}{3}$

$y = \frac{5}{3}x - 5$

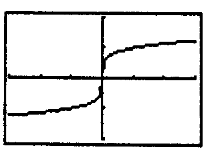
13. The area is $A = \pi r^2$ and the circumference is $C = 2\pi r$. Thus, $r = \frac{C}{2\pi} \Rightarrow A = \pi\left(\frac{C}{2\pi}\right)^2 = \frac{C^2}{4\pi}$.

14. The surface area is $S = 4\pi r^2 \Rightarrow r = \left(\frac{S}{4\pi}\right)^{1/2}$. The volume is $V = \frac{4}{3}\pi r^3 \Rightarrow r = \sqrt[3]{\frac{3V}{4\pi}}$. Substitution into the formula for surface area gives $S = 4\pi r^2 = 4\pi\left(\frac{3V}{4\pi}\right)^{2/3}$.

15. The coordinates of a point on the parabola are (x, x^2). The angle of inclination θ joining this point to the origin satisfies the equation $\tan\theta = \frac{x^2}{x} = x$. Thus the point has coordinates $(x, x^2) = (\tan\theta, \tan^2\theta)$.

16. $\tan\theta = \frac{\text{rise}}{\text{run}} = \frac{h}{500} \Rightarrow h = 500 \tan\theta$ ft.

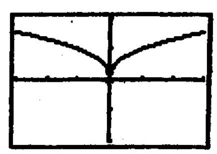
17.



[−3, 3] by [−2, 2]

Symmetric about the origin.

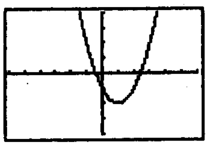
18.



[−3, 3] by [−2, 2]

Symmetric about the y-axis.

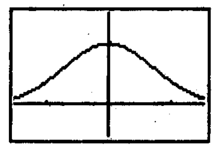
19.



[−6, 6] by [−4, 4]

Neither

20.



[−1.5, 1.5] by [−0.5, 1.5]

Symmetric about the y-axis.

21. $y(-x) = (-x)^2 + 1 = x^2 + 1 = y(x)$
Even

22. $y(-x) = (-x)^5 - (-x) = -x^5 + x^3 + x = -y(x)$
Odd

23. $y(-x) = 1 - \cos(-x) = 1 - \cos x = y(x)$
Even

24. $y(-x) = \sec(-x)\tan(-x)$
$= \frac{\sin(-x)}{\cos^2(-x)} = \frac{-\sin x}{\cos^2 x}$
$= -\sec x \tan x = -y(x)$
Odd

25. $y(-x) = \frac{(-x)^4 + 1}{(-x)^3 - 2(-x)} = \frac{x^4 + 1}{-x^3 + 2x} = -\frac{x^4 + 1}{x^3 - 2x} = -y(x)$
Odd

26. $y(-x) = 1 - \sin(-x) = 1 + \sin x$
Neither even nor odd

27. $y(-x) = -x + \cos(-x) = -x + \cos x$
Neither even nor odd

28. $y(-x) = \sqrt{(-x)^4 - 1} = \sqrt{x^4 - 1} = y(x)$
Even

29. (a) The function is defined for all values of x, so the domain is $(-\infty, \infty)$.
(b) Since $|x|$ attains all nonnegative values, the range is $[-2, \infty)$.

30. (a) Since the square root requires $1 - x \geq 0$, the domain is $(-\infty, 1]$.
(b) Since $\sqrt{1-x}$ attains all nonnegative values, the range is $[-2, \infty)$.

31. (a) Since the square root requires $16 - x^2 \geq 0$, the domain is $[-4, 4]$
(b) For values of x in the domain, $0 \leq 16 - x^2 \leq 16$, so $0 \leq \sqrt{16 - x^2} \leq 4$. The range is $[0, 4]$.

32. (a) The function is defined for all value os x, so the domain is $(-\infty, \infty)$.
(b) Since 3^{2-x} attains all possible values, the range is $(1, \infty)$.

33. (a) The function is defined for all values of x, so the domain is $(-\infty, \infty)$.
(b) Since $2e^{-x}$ attains all positive values, the range is $(-3, \infty)$.

34. (a) The function is equivalent to $y = \tan 2x$, so we require $2x \neq \frac{k\pi}{2}$ for odd integers k. The domain is given by $x \neq \frac{k\pi}{4}$ for odd integers k.
(b) Since the tangent function attains all values, the range is $(-\infty, \infty)$.

35. (a) The function is defined for all values of x, so the domain is $(-\infty, \infty)$.
(b) The sine function attains values from -1 to 1, so $-2 \leq 2\sin(3x+\pi) \leq 2$, and hence $-3 \leq 2\sin(3x+\pi) - 1 \leq 1$. The range is $[-3, 1]$.

36. (a) The function is defined for all values of x, so the domain is $(-\infty, \infty)$.
(b) The function is equivalent to $y = \sqrt[5]{x^2}$, which attains all nonnegative values. The range is $[0, \infty)$.

37. (a) The logarithm requires $x - 3 > 0$, so the domain is $(3, \infty)$.
(b) The logarithm attains all real values, so the range is $(-\infty, \infty)$.

38. (a) The function is defined for all values of x, so the domain is $(-\infty, \infty)$.
(b) The cube root attains all real values, so the range is $(-\infty, \infty)$.

39. (a) The function is defined for $-4 \leq x \leq 4$, so the domain is $[-4, 4]$.
(b) The function is equivalent to $y = \sqrt{|x|}$, $-4 \leq x \leq 4$, which attains values from 0 to 2 for x in the domain. The range is $[0, 2]$.

40. (a) The function is defined for $-2 \leq x \leq 2$, so the domain is $[-2, 2]$.
(b) The range is $[-1, 1]$.

41. First piece: Line through $(0, 1)$ and $(1, 0)$

$m = \frac{0-1}{1-0} = \frac{-1}{1} = -1$

$y = -x + 1$ or $1 - x$

Second piece: Line through $(1, 1)$ and $(2, 0)$

$m = \frac{0-1}{2-1} = \frac{-1}{1} = -1$

$y = -(x-1) + 1$

$y = -x + 2$ or $2 - x$

$$f(x) = \begin{cases} 1 - x, & 0 \leq x < 1 \\ 2 - x, & 1 \leq x \leq 2 \end{cases}$$

42. First piece: Line through $(0, 0)$ and $(2, 5)$

$m = \frac{5-0}{2-0} = \frac{5}{2}$

$y = \frac{5}{2}x$

Second piece: Line through $(2, 5)$ and $(4, 0)$

$m = \frac{0-5}{4-2} = \frac{-5}{2} = -\frac{5}{2}$

$y = -\frac{5}{2}(x-2) + 5$

$y = -\frac{5}{2}x + 10$ or $10 - \frac{5x}{2}$

$$f(x) = \begin{cases} \frac{5x}{2}, & 0 \leq x < 2 \\ 10 - \frac{5x}{2}, & 2 \leq x \leq 4 \end{cases}$$

(Note: $x = 2$ can be included on either piece.)

43. (a) $(f \circ g)(-1) = f(g(-1)) = f\left(\frac{1}{\sqrt{-1+2}}\right) = f(1) = \frac{1}{1} = 1$

(b) $(g \circ f)(2) = g(f(2)) = g\left(\frac{1}{2}\right) = \frac{1}{\sqrt{1/2+2}} = \frac{1}{\sqrt{2.5}}$ or $\sqrt{\frac{2}{5}}$

(c) $(f \circ f)(x) = f(f(x)) = f\left(\frac{1}{x}\right) = \frac{1}{1/x} = x,\ x \neq 0$

(d) $(g \circ g)(x) = g(g(x)) = g\left(\frac{1}{\sqrt{x+2}}\right) = \frac{1}{\sqrt{1/\sqrt{x+2}+2}}$

$$= \frac{\sqrt[4]{x+2}}{\sqrt{1+2\sqrt{x+2}}}$$

44. (a) $(f \circ g)(-1) = f(g(-1))$

$= f(\sqrt[3]{-1+1})$

$= f(0) = 2 - 0 = 2$

(b) $(g \circ f)(2)g(f(2)) = g(2-2) = g(0) = \sqrt[3]{0+1} = 1$

(c) $(f \circ f)(x) = f(f(x)) = f(2-x) = 2-(2-x) = x$

(d) $(g \circ g)(x) = g(g(x)) = g(\sqrt[3]{x+1}) = \sqrt[3]{\sqrt[3]{x+1}+1}$

45. (a) $(f \circ g)(x) = f(g(x))$

$= f(\sqrt{x+2})$

$= 2 - (\sqrt{x+2})^2$

$= -x,\ x \geq -2$

$(g \circ f)(x) = g(f(x))$

$= g(2-x^2)$

$= \sqrt{(2-x^2)+2} = \sqrt{4-x^2}$

(b) Domain of $f \circ g$: $[-2, \infty)$

Domain of $g \circ f$: $[-2, 2]$

(c) Range of $f \circ g$: $(-\infty, 2]$

Range of $g \circ f$: $[0, 2]$

46. (a) $(f \circ g)(x) = f(g(x))$

$= f(\sqrt{1-x})$

$= \sqrt{\sqrt{1-x}}$

$= \sqrt[4]{1-x}$

$(g \circ f)(x) = g(f(x)) = g(\sqrt{x}) = \sqrt{1 - \sqrt{x}}$

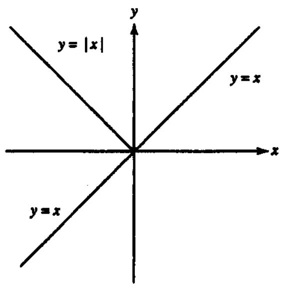
(b) Domain of $f \circ g$: $(-\infty, 1]$

Domain of $g \circ f$: $[0, 1]$

(c) Range of $f \circ g$: $[0, \infty)$

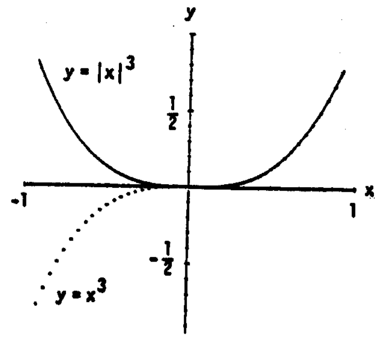
Range of $g \circ f$: $[0, 1]$

47.



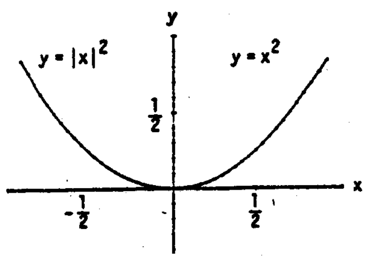
The graph of $f_2(x) = f_1(|x|)$ is the same as the graph of $f_1(x)$ to the right of the y-axis. The graph of $f_2(x)$ to the left of the y-axis is the reflection of $y = f_1(x)$, $x \geq 0$ across the y-axis.

48.



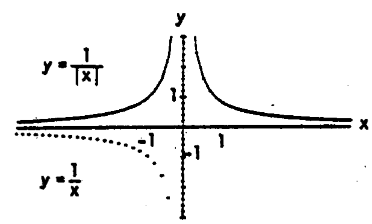
The graph of $f_2(x) = f_1(|x|)$ is the same as the graph of $f_1(x)$ to the right of the y-axis. The graph of $f_2(x)$ to the left of the y-axis is the reflection of $y = f_1(x)$, $x \geq 0$ across the y-axis.

49.



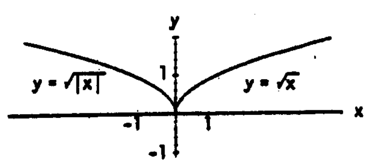
It does not change the graph.

50.



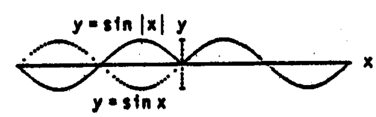
The graph of $f_2(x) = f_1(|x|)$ is the same as the graph of $f_1(x)$ to the right of the y-axis. The graph of $f_2(x)$ to the left of the y-axis is the reflection of $y = f_1(x)$, $x \geq 0$ across the y-axis.

51.



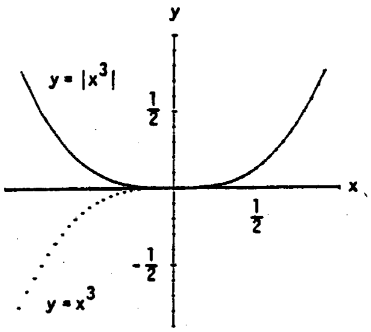
The graph of $f_2(x) = f_1(|x|)$ is the same as the graph of $f_1(x)$ to the right of the y-axis. The graph of $f_2(x)$ to the left of the y-axis is the reflection of $y = f_1(x)$, $x \geq 0$ across the y-axis.

52.



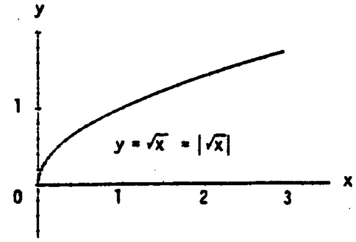
The graph of $f_2(x) = f_1(|x|)$ is the same as the graph of $f_1(x)$ to the right of the y-axis. The graph of $f_2(x)$ to the left of the y-axis is the reflection of $y = f_1(x)$, $x \geq 0$ across the y-axis.

53.



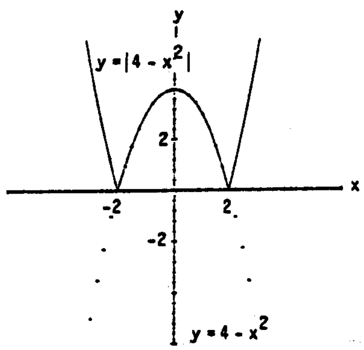
Whenever $g_1(x)$ is positive, the graph of $y = g_2(x) = |g_1(x)|$ is the same as the graph of $y = g_1(x)$. When $g_1(x)$ is negative, the graph of $y = g_2(x)$ is the reflection of the graph of $y = g_1(x)$ across the x-axis.

54.



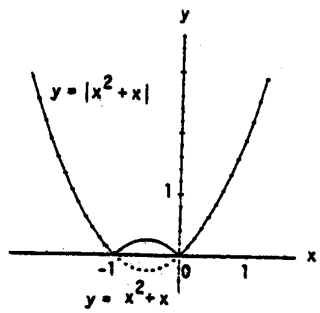
It does not change the graph.

55.



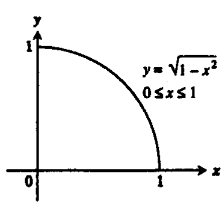
Whenever $g_1(x)$ is positive, the graph of $y = g_2(x) = |g_1(x)|$ is the same as the graph of $y = g_1(x)$. When $g_1(x)$ is negative, the graph of $y = g_2(x)$ is the reflection of the graph of $y = g_1(x)$ across the x-axis.

56.



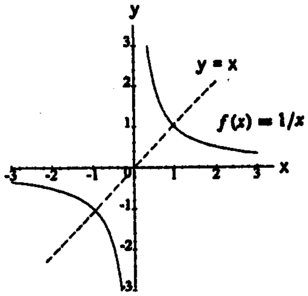
Whenever $g_1(x)$ is positive, the graph of $y = g_2(x) = |g_1(x)|$ is the same as the graph of $y = g_1(x)$. When $g_1(x)$ is negative, the graph of $y = g_2(x)$ is the reflection of the graph of $y = g_1(x)$ across the x-axis.

57. (a) The graph is symmetric about $y = x$.



(b) $y = \sqrt{1-x^2} \Rightarrow y^2 = 1 - x^2 \Rightarrow x^2 = 1 - y^2 \Rightarrow x = \sqrt{1-y^2} \Rightarrow y = \sqrt{1-x^2} = f^{-1}(x)$

58. The graph is symmetric about $y = x$.



(b) $y = \frac{1}{x} \Rightarrow x = \frac{1}{y} \Rightarrow y = \frac{1}{x} = f^{-1}(x)$

59. (a) $y = 2 - 3x \rightarrow 3x = 2 - y \rightarrow x = \frac{2-y}{3}$.

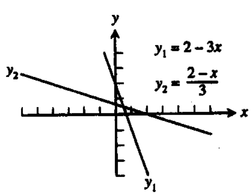
Interchange x and y: $y = \frac{2-x}{3} \rightarrow f^{-1}(x) = \frac{2-x}{3}$

Verify.

$(f \circ f^{-1})(x) = f(f^{-1}(x)) = f\left(\frac{2-x}{3}\right) = 2 - 3\left(\frac{2-x}{3}\right) = 2 - (2 - x) = x$

$(f^{-1} \circ f)(x) = f^{-1}(f(x)) = f^{-1}(2 - 3x) = \frac{2 - (2 - 3x)}{3} = \frac{3x}{3} = x$

(b)



60. (a) $y = (x + 2)^2,\ x \geq -2 \rightarrow \sqrt{y} = x + 2 \rightarrow x = \sqrt{y} - 2$.

Interchange x and y: $y = \sqrt{x} - 2 \rightarrow f^{-1}(x) = \sqrt{x} - 2$

Verify.

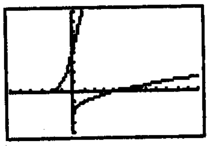
For $x \geq 0$ (the domain of f^{-1})

$(f \circ f^{-1})(x) = f(f^{-1}(x)) = f(\sqrt{x} - 2) = [(\sqrt{x} - 2) + 2]^2 = (\sqrt{x})^2 = x$

For $x \geq -2$ (the domain of f),

$(f^{-1} \circ f)(x) = f^{-1}(f(x)) = f^{-1}((x + 2)^2) = \sqrt{(x + 2)^2} - 2 = |x + 2| - 2 = (x + 2) - 2 = x$

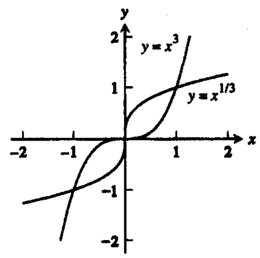
(b)



[−6, 12] by [−4, 8]

61. (a) $f(g(x)) = \left(\sqrt[3]{x}\right)^3 = x$, $g(f(x)) = \sqrt[3]{x^3} = x$

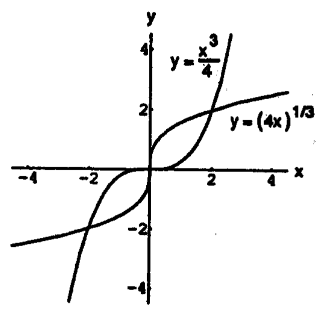
(b)



62. (a) $h(k(x)) = \frac{1}{4}\left((4x)^{1/3}\right)^3 = x$,

$k(h(x)) = \left(4 \cdot \frac{x^3}{4}\right)^{1/3} = x$

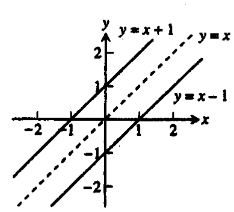
(b)



63. (a) $y = x + 1 \Rightarrow x = y - 1 \Rightarrow f^{-1}(x) = x - 1$

(b) $y = x + b \Rightarrow x = y - b \Rightarrow f^{-1}(x) = x - b$

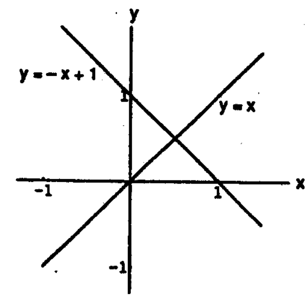
(c) Their graphs will be parallel to one another and lie on opposite sides of the line $y = x$ equidistant from that line.



64. (a) $y = -x + 1 \Rightarrow x = -y + 1 \Rightarrow f^{-1}(x) = 1 - x$;
the lines intersect at a right angle

(b) $y = -x + b \Rightarrow x = -y + b \Rightarrow f^{-1}(x) = b - x$;
the lines intersect at a right angle

(c) f is its own inverse



65. $x = 2.71828182846$ (using a TI-92 Plus calculator).

66. $e^{\ln x} = x$ and $\ln(e^x) = x$ for all $x > 0$

67. (a) $e^{\ln 7.2} = 7.2$ (b) $e^{-\ln x^2} = \frac{1}{e^{\ln x^2}} = \frac{1}{x^2}$ (c) $e^{\ln x - \ln y} = e^{\ln(x/y)} = \frac{x}{y}$

68. (a) $e^{\ln(x^2+y^2)} = x^2 + y^2$ (b) $e^{-\ln 0.3} = \frac{1}{e^{\ln 0.3}} = \frac{1}{0.3} = \frac{10}{3}$ (c) $e^{\ln \pi x - \ln 2} = e^{\ln(\pi x/2)} = \frac{\pi x}{2}$

69. (a) $2 \ln \sqrt{e} = 2 \ln e^{1/2} = (2)\left(\frac{1}{2}\right) \ln e = 1$ (b) $\ln(\ln e^e) = \ln(e \ln e) = \ln e = 1$

(c) $\ln e^{(-x^2-y^2)} = (-x^2 - y^2) \ln e = -x^2 - y^2$

70. (a) $\ln(e^{\sec \theta}) = (\sec \theta)(\ln e) = \sec \theta$ (b) $\ln e^{(e^x)} = (e^x)(\ln e) = e^x$

(c) $\ln(e^{2 \ln x}) = \ln\left(e^{\ln x^2}\right) = \ln x^2 = 2 \ln x$

71. Using a calculator, $\sin^{-1}(0.6) \approx 0.6435$ radians or $36.8699°$.

72. Using a calculator, $\tan^{-1}(-2.3) \approx -1.1607$ radians or $-66.5014°$.

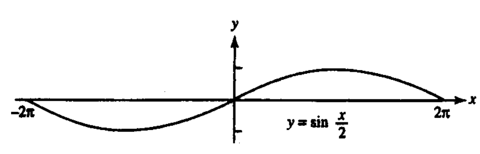
73. Since $\cos \theta = \frac{3}{7}$ and $0 \le \theta \le \pi$, $\sin \theta = \sqrt{1 - \cos^2 \theta} = \sqrt{1 - \left(\frac{3}{7}\right)^2} = \sqrt{\frac{40}{49}} = \frac{\sqrt{40}}{7}$. Therefore,

$\sin \theta = \frac{\sqrt{40}}{7}$, $\cos \theta = \frac{3}{7}$, $\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{\sqrt{40}}{3}$, $\cot \theta = \frac{1}{\tan \theta}, = \frac{3}{\sqrt{40}}$, $\sec \theta = \frac{1}{\cos \theta} = \frac{7}{3}$, $\csc \theta = \frac{1}{\sin \theta} = \frac{7}{\sqrt{40}}$

74. (a) Note that $\sin^{-1}(-0.2) \approx -0.2014$. In $[0, 2\pi)$, the solutions are $x = \pi - \sin^{-1}(-0.2) \approx 3.3430$ and $x = \sin^{-1}(-0.2) + 2\pi \approx 6.0818$.

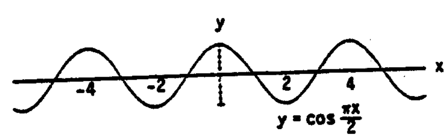
(b) Since the period of sin x is 2π, the solutions are $x \approx 3.3430 + 2k\pi$ and $x \approx 6.0818 + 2k\pi$, k any integer.

75.



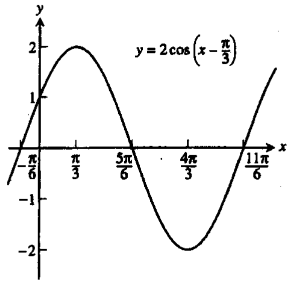
period $= 4\pi$

76.



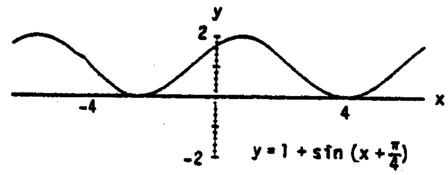
period $= 4$

77.



period $= 2\pi$

78.



period $= 2\pi$

79. (a) $\sin B = \sin \frac{\pi}{3} = \frac{b}{c} = \frac{b}{2} \Rightarrow b = 2 \sin \frac{\pi}{3} = 2\left(\frac{\sqrt{3}}{2}\right) = \sqrt{3}$. By the theorem of Pythagoras,

$a^2 + b^2 = c^2 \Rightarrow a = \sqrt{c^2 - b^2} = \sqrt{4-3} = 1.$

(b) $\sin B = \sin \frac{\pi}{3} = \frac{b}{c} = \frac{2}{c} \Rightarrow c = \frac{2}{\sin \frac{\pi}{3}} = \frac{2}{\left(\frac{\sqrt{3}}{2}\right)} = \frac{4}{\sqrt{3}}$. Thus, $a = \sqrt{c^2 - b^2} = \sqrt{\left(\frac{4}{\sqrt{3}}\right)^2 - (2)^2} = \sqrt{\frac{4}{3}} = \frac{2}{\sqrt{3}}$.

80. (a) $\sin A = \frac{a}{c} \Rightarrow a = c \sin A$

(b) $\tan A = \frac{a}{b} \Rightarrow a = b \tan A$

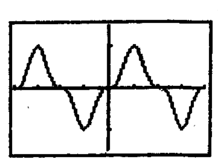
81. (a) $\tan B = \frac{b}{a} \Rightarrow a = \frac{b}{\tan B}$

(b) $\sin A = \frac{a}{c} \Rightarrow c = \frac{a}{\sin A}$

82. (a) $\sin A = \frac{a}{c}$

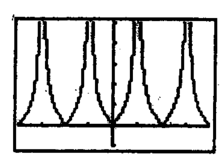
(c) $\sin A = \frac{a}{c} = \frac{\sqrt{c^2 - b^2}}{c}$

83. Since sin x has period 2π, $\sin^3(x + 2\pi) = \sin^3(x)$. This function has period 2π. A graph shows that no smaller number works for the period.



$[-2\pi, 2\pi]$ by $[-1.5, 1.5]$

84. Since tan x has period π, $|\tan(x + \pi)| = |\tan x|$. This function has period π. A graph shows that no smaller number works for the period.



$[-2\pi, 2\pi]$ by $[-1, 5]$

85. $\cos\left(x + \frac{\pi}{2}\right) = \cos x \cos\left(\frac{\pi}{2}\right) - \sin x \sin\left(\frac{\pi}{2}\right) = (\cos x)(0) - (\sin x)(1) = -\sin x$

86. $\sin\left(x - \frac{\pi}{2}\right) = \sin x \cos\left(-\frac{\pi}{2}\right) + \cos x \sin\left(-\frac{\pi}{2}\right) = (\sin x)(0) + (\cos x)(-1) = -\cos x$

87. $\sin \frac{7\pi}{12} = \sin\left(\frac{\pi}{4} + \frac{\pi}{3}\right) = \sin \frac{\pi}{4} \cos \frac{\pi}{3} + \cos \frac{\pi}{4} \sin \frac{\pi}{3} = \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{6} + \sqrt{2}}{4}$

88. $\cos \frac{11\pi}{12} = \cos\left(\frac{\pi}{4} + \frac{2\pi}{3}\right) = \cos \frac{\pi}{4} \cos \frac{2\pi}{3} - \sin \frac{\pi}{4} \sin \frac{2\pi}{3} = \left(\frac{\sqrt{2}}{2}\right)\left(-\frac{1}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = -\frac{\sqrt{2} + \sqrt{6}}{4}$

89. (a) $\frac{\pi}{6}$ (b) $-\frac{\pi}{4}$ (c) $\frac{\pi}{3}$

90. (a) $\frac{2\pi}{3}$ (b) $\frac{\pi}{4}$ (c) $\frac{5\pi}{6}$

91. (a) $\frac{\pi}{4}$ (b) $\frac{5\pi}{6}$ (c) $\frac{\pi}{3}$

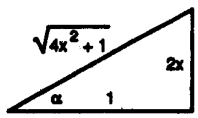
92. (a) $\frac{\pi}{4}$ (b) $\frac{5\pi}{6}$ (c) $\frac{\pi}{3}$

93. $\sec\left(\cos^{-1}\frac{1}{2}\right) = \sec\left(\frac{\pi}{3}\right) = 2$

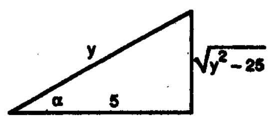
94. $\cot\left(\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right) = \cot\left(-\frac{\pi}{3}\right) = -\frac{1}{\sqrt{3}}$

95. $\tan\left(\sec^{-1}1\right) + \sin\left(\csc^{-1}(-2)\right) = \tan\left(\cos^{-1}\frac{1}{1}\right) + \sin\left(\sin^{-1}\left(-\frac{1}{2}\right)\right) = \tan(0) + \sin\left(-\frac{\pi}{6}\right) = 0 + \left(-\frac{1}{2}\right) = -\frac{1}{2}$

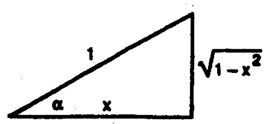
96. $\sec\left(\tan^{-1}1 + \csc^{-1}1\right) = \sec\left(\frac{\pi}{4} + \sin^{-1}\frac{1}{1}\right) = \sec\left(\frac{\pi}{4} + \frac{\pi}{2}\right) = \sec\left(\frac{3\pi}{4}\right) = -\sqrt{2}$

97. $\alpha = \tan^{-1}2x$ indicates the diagram 

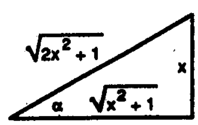
$\Rightarrow \sec\left(\tan^{-1}2x\right) = \sec\alpha = \sqrt{4x^2+1}$

98. $\alpha = \sec^{-1}\frac{y}{5}$ indicates the diagram 

$\Rightarrow \tan\left(\sec^{-1}\frac{y}{5}\right) = \tan\alpha = \frac{\sqrt{y^2-25}}{5}$

99. $\alpha = \cos^{-1}x$ indicates the diagram 

$\Rightarrow \tan\left(\cos^{-1}x\right) = \tan\alpha = \frac{\sqrt{1-x^2}}{x}$

100. $\alpha = \tan^{-1}\frac{x}{\sqrt{x^2+1}}$ indicates the diagram 

$\Rightarrow \sin\left(\tan^{-1}\frac{x}{\sqrt{x^2+1}}\right) = \sin\alpha = \frac{x}{\sqrt{2x^2+1}}$

101. (a) Defined; there is an angle whose tangent is 2.
(b) Not defined; there is no angle whose cosine is 2.

102. (a) Not defined; there is no angle whose cosecant is $\frac{1}{2}$.
(b) Defined; there is an angle whose cosecant is 2.

103. (a) Not defined; there is no angle whose secant is 0.
(b) Not defined; there is no angle whose sine is $\sqrt{2}$.

104. (a) Defined; there is an angle whose cotangent is $-\frac{1}{2}$.

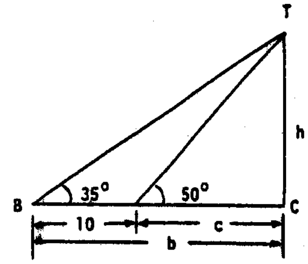
(b) Not defined; there is no angle whose cosine is -5.

105. Let h = height of vertical pole, and let b and c denote the distances of points B and C from the base of the pole, measured along the flat ground, respectively. Then, $\tan 50^\circ = \frac{h}{c}$, $\tan 35^\circ = \frac{h}{b}$, and $b - c = 10$. Thus, $h = c \tan 50^\circ$ and $h = b \tan 35^\circ = (c + 10) \tan 35^\circ$

$\Rightarrow c \tan 50^\circ = (c + 10) \tan 35^\circ \Rightarrow c\,(\tan 50^\circ - \tan 35^\circ) = 10 \tan 35^\circ$

$\Rightarrow c = \frac{10 \tan 35^\circ}{\tan 50^\circ - \tan 35^\circ} \Rightarrow h = c \tan 50^\circ = \frac{10 \tan 35^\circ \tan 50^\circ}{\tan 50^\circ - \tan 35^\circ}$

≈ 16.98 m.



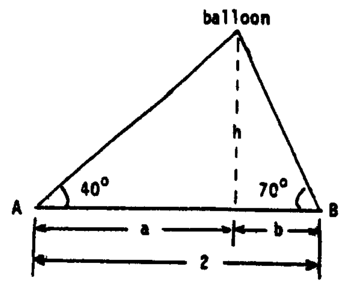
106. Let h = height of balloon above ground. From the figure at the right, $\tan 40^\circ = \frac{h}{a}$, $\tan 70^\circ = \frac{h}{b}$, and $a + b = 2$. Thus,

$h = b \tan 70^\circ \Rightarrow h = (2 - a) \tan 70^\circ$ and $h = a \tan 40^\circ$

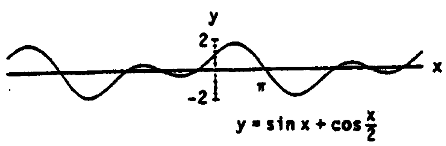
$\Rightarrow (2 - a) \tan 70^\circ = a \tan 40^\circ \Rightarrow a(\tan 40^\circ + \tan 70^\circ) = 2 \tan 70^\circ$

$\Rightarrow a = \frac{2 \tan 70^\circ}{\tan 40^\circ + \tan 70^\circ} \Rightarrow h = a \tan 40^\circ = \frac{2 \tan 70^\circ \tan 40^\circ}{\tan 40^\circ + \tan 70^\circ}$

≈ 1.3 km.



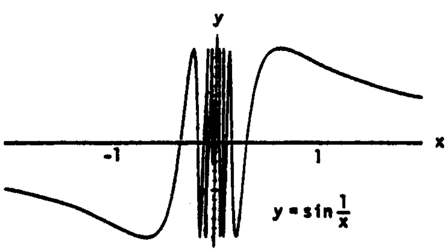
107. (a)



(b) The period appears to be 4π.

(c) $f(x + 4\pi) = \sin(x + 4\pi) + \cos\left(\frac{x + 4\pi}{2}\right) = \sin(x + 2\pi) + \cos\left(\frac{x}{2} + 2\pi\right) = \sin x + \cos \frac{x}{2}$ since the period of sine and cosine is 2π. Thus, $f(x)$ has period 4π.

108. (a)



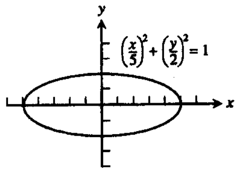
(b) $D = (-\infty, 0) \cup (0, \infty)$; $R = [-1, 1]$

(c) f is not periodic. Suppose f has period p. Then $f\left(\frac{1}{2\pi} + kp\right) = f\left(\frac{1}{2\pi}\right) = \sin 2\pi = 0$ for all integers k. Choose k so large that $\frac{1}{2\pi} + kp > \frac{1}{\pi} \Rightarrow 0 < \frac{1}{(1/2\pi) + kp} < \pi$. But then $f\left(\frac{1}{2\pi} + kp\right) = \sin\left(\frac{1}{(1/2\pi) + kp}\right) > 0$

which is a contradiction. Thus f has no period, as claimed.

109. (a) Substituting $\cos t = \frac{x}{5}$ and $\sin t = \frac{y}{2}$ in the identity $\cos^2 t + \sin^2 t = 1$ gives the Cartesian equation $\left(\frac{x}{5}\right)^2 + \left(\frac{y}{2}\right)^2 = 1$. The entire ellipse is traced by the curve.

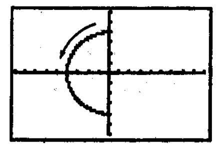
(b)



Initial point: $(5, 0)$
Terminal point: $(5, 0)$
The ellipse is traced exactly once in a counterclockwise direction starting and ending at the point $(5, 0)$.

110. (a) Substituting $\cos t = \frac{x}{4}$ and $\sin t = \frac{y}{4}$ in the identity $\cos^2 t + \sin^2 t = 1$ gives the Cartesian equation $\left(\frac{x}{4}\right)^2 + \left(\frac{y}{4}\right)^2 = 1$, or $x^2 + y^2 = 16$. The left half of the circle is traced by the parametrized curve.

(b)

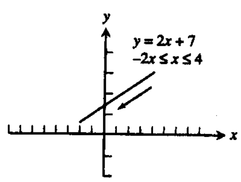


[−9, 9] by [−6, 6]

Initial point: $(0, 4)$
Terminal point: None (since the endpoint $\frac{3\pi}{2}$ is not included in the t-interval)
The semicircle is traced in a counterclockwise direction starting at $(0, 4)$ and extending to, but not including, $(0, -4)$.

111. (a) Substituting $t = 2 - x$ into $y = 11 - 2t$ gives the Cartesian equation $y = 11 - 2(2 - x)$, or $y = 2x + 7$. The part of the line from $(4, 15)$ to $(-2, 3)$ is traced by the parametrized curve.

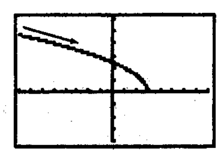
(b)



Initial point: $(4, 15)$
Terminal point: $(-2, 3)$
The line segment is traced from right to left starting at $(4, 15)$ and ending at $(-2, 3)$.

112. (a) Substituting $t = x - 1$ into $y = (t - 1)^2$ gives the Cartesian equation $y = (x - 1 - 1)^2$, or $y = (x - 2)^2$. The part of the parabola for $x \le 2$ is traced by the parametrized curve.

(b)



[−8, 8] by [−4, 6]

Initial point: None
Terminal point: $(3, 0)$
The curve is traced from left to right ending at the point $(3, 0)$.

113. (a) For simplicity, we assume that x and y are linear functions of t, and that the point (x, y) starts at $(-2, 5)$ for $t = 0$ and ends at $(4, 3)$ for $t = 1$. Then $x = f(t)$, where $f(0) = -2$ and $f(1) = 4$. Since slope $= \frac{\Delta x}{\Delta t} = \frac{4 - (-2)}{1 - 0} = 6$, $x = f(t) = 6t - 2 = -2 + 6t$. Also, $y = g(t)$, where $g(0) = 5$ and $g(1) = 3$.

Since slope $= \frac{\Delta y}{\Delta t} = \frac{3-5}{1-0} = -2$, $y = g(t) = -2t + 5 = 5 - 2t$. One possible parametrization is: $x = -2 + 6t$, $y = 5 - 2t$, $0 \le t \le 1$.

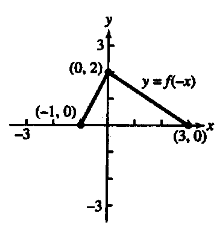
114. For simplicity, we assume that x and y are linear functions of t, and that the point (x, y) passes through $(-3, -2)$ for $t = 0$ and $(4, -1)$ for $t = 1$. Then $x = f(t)$, where $f(0) = -3$ and $f(1) = 4$. Since slope $= \frac{\Delta x}{\Delta t} = \frac{4-(-3)}{1-0} = 7$, $x = f(t) = 7t - 3 = -3 + 7t$. Also, $y = g(t)$, where $g(0) = -2$ and $g(1) = -1$. Since slope $= \frac{\Delta y}{\Delta t} = \frac{-1-(-2)}{1-0} = 1$, $y = g(t) = t - 2 = -2 + t$. One possible parametrization is: $x = -3 + 7t$, $y = -2 + t$, $-\infty < t < \infty$.

115. For simplicity, we assume that x and y are linear functions of t, and that the point (x, y) starts at $(2, 5)$ for $t = 0$ and passes through $(-1, 0)$ for $t = 1$. Then $x = f(t)$, where $f(0) = 2$ and $f(1) = -1$. Since slope $= \frac{\Delta x}{\Delta t} = \frac{-1-2}{1-0} = -3$, $x = f(t) = -3t + 2 = 2 - 3t$. Also, $y = g(t)$, where $g(0) = 5$ and $g(1) = 0$. Since slope $= \frac{\Delta y}{\Delta t} = \frac{0-5}{1-0} = -5$, $y = g(t) = -5t + 5 = 5 - 5t$. One possible parametrization is: $x = 2 - 3t$, $y = 5 - 5t$, $t \ge 0$.

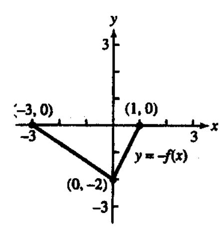
116. One possible parametrization is: $x = t$, $y = t(t-4)$, $t \le 2$.

PRELIMINARY CHAPTER ADDITIONAL EXERCISES–THEORY, EXAMPLES, APPLICATIONS

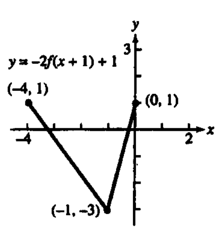
1. (a) The given graph is reflected about the y-axis.



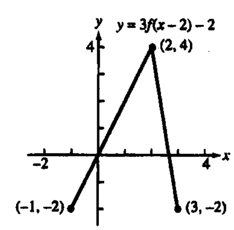
(b) The given graph is reflected about the x-axis.



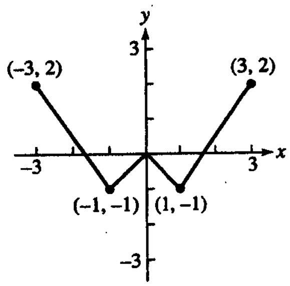
(c) The given graph is shifted left 1 unit, stretched vertically by a factor of 2, reflected about the x-axis, and then shifted upward 1 unit.



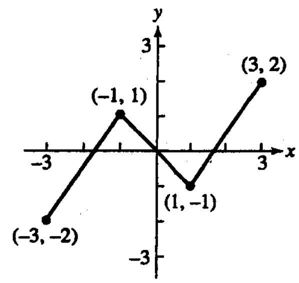
(d) The given graph is shifted right 2 units, stretched vertically by a factor of 3, and then shifted downward 2 units.



2. (a)

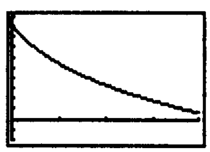


(b)



3. (a) $y = 100{,}000 - 10{,}000x,\ 0 \le x \le 10$
   (b)
$$\begin{aligned} y &= 55{,}000 \\ 100{,}000 - 10{,}000x &= 55{,}000 \\ -10{,}000x &= -45{,}000 \\ x &= 4.5 \end{aligned}$$
   The value is \$55,000 after 4.5 years.

4. (a) $f(0) = 90$ units
   (b) $f(2) = 90 - 52 \ln 3 \approx 32.8722$ units
   (c)

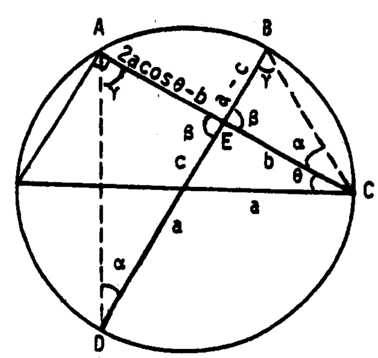


[0, 4] by [−20, 100]

5. $1500(1.08)^t = 5000 \to 1.08^t = \frac{5000}{1500} = \frac{10}{3} \to \ln(1.08)^t = \ln \frac{10}{3} \to t \ln 1.08 = \ln \frac{10}{3} \to t = \frac{\ln(10/3)}{\ln 1.08} \approx 15.6439$

   It will take about 15.6439 years. (If the bank only pays interest at the end of the year, it will take 16 years.)

6. The angles labeled γ in the accompanying figure are equal since both angles subtend arc CD. Similarly, the two angles labeled α are equal since they both subtend arc AB. Thus, triangles AED and BEC are similar which implies $\frac{a-c}{b} = \frac{2a\cos\theta - b}{a+c} \Rightarrow (a-c)(a+c) = b(2a\cos\theta - b)$ $\Rightarrow a^2 - c^2 = 2ab\cos\theta - b^2 \Rightarrow c^2 = a^2 + b^2 - 2ab\cos\theta$.



7. As in the proof of the law of sines of Section P.5, Exercise 35, $ah = bc \sin A = ab \sin C = ac \sin B$ $\Rightarrow$ the area of ABC $= \frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}ah = \frac{1}{2}bc \sin A = \frac{1}{2}ab \sin C = \frac{1}{2}ac \sin B$.

8. (a) The coordinates of P are $\left(\frac{a+0}{2}, \frac{b+0}{2}\right) = \left(\frac{a}{2}, \frac{b}{2}\right)$. Thus the slope of OP $= \frac{\Delta y}{\Delta x} = \frac{b/2}{a/2} = \frac{b}{a}$.

   (b) The slope of AB $= \frac{b-0}{0-a} = -\frac{b}{a}$. The line segments AB and OP are perpendicular when the product of their slopes is $-1 = \left(\frac{b}{a}\right)\left(-\frac{b}{a}\right) = -\frac{b^2}{a^2}$. Thus, $b^2 = a^2 \Rightarrow a = b$ (since both are positive). Therefore, AB is perpendicular to OP when $a = b$.

9. Triangle ABD is an isosceles right triangle with its right angle at B and an angle of measure $\frac{\pi}{4}$ at A. We therefore have $\frac{\pi}{4} = \angle DAB = \angle DAE + \angle CAB = \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{2}$.

10. $\ln x^{(x^x)} = x^x \ln x$ and $\ln\left(x^x\right)^x = x \ln x^x = x^2 \ln x$; then, $x^x \ln x = x^2 \ln x \Rightarrow x^x = x^2 \Rightarrow x \ln x = 2 \ln x$ $\Rightarrow x = 2$. Therefore, $x^{(x^x)} = \left(x^x\right)^x$ when $x = 2$.

11. (a) If f is even, then $f(x) = f(-x)$ and $h(-x) = g(f(-x)) = g(f(x)) = h(x)$.
    If f is odd, then $f(-x) = -f(x)$ and $h(-x) = g(f(-x)) = g(-f(x)) = g(f(x)) = h(x)$ because g is even.
    If f is neither, then h may not be even. For example, if $f(x) = x^2 + x$ and $g(x) = x^2$, then $h(x) = x^4 + 2x^3 + x^2$ and $h(-x) = x^4 - 2x^3 + x^2 \neq h(x)$. Therefore, h need not be even.

    (b) No, h is not always odd. Let $g(t) = t$ and $f(x) = x^2$. Then, $h(x) = g(f(x)) = f(x) = x^2$ is even although g is odd.
    If f is odd, then $f(-x) = -f(x)$ and $h(-x) = g(f(-x)) = g(-f(x)) = -g(f(x)) = -h(x)$ because g is odd. In this case, h is odd. However, if f is even, as in the above counterexample, we see that h need not be odd.

12. $A(t) = A_0e^{rt}$; $A(t) = 2A_0 \Rightarrow 2A_0 = A_0e^{rt} \Rightarrow e^{rt} = 2 \Rightarrow rt = \ln 2 \Rightarrow t = \frac{\ln 2}{r} \Rightarrow t \approx \frac{.7}{r} = \frac{70}{100r} = \frac{70}{(r\%)}$

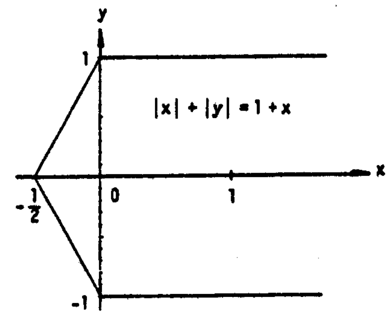
13. There are (infinitely) many such function pairs. For example, $f(x) = 3x$ and $g(x) = 4x$ satisfy $f(g(x)) = f(4x) = 3(4x) = 12x = 4(3x) = g(3x) = g(f(x))$.

14. Yes, there are many such function pairs. For example, if $g(x) = (2x+3)^3$ and $f(x) = x^{1/3}$, then $(f \circ g)(x) = f(g(x)) = f\left((2x+3)^3\right) = \left((2x+3)^3\right)^{1/3} = 2x + 3$.

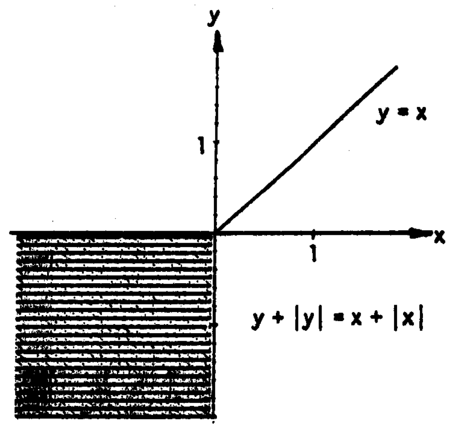
15. If f is odd and defined at x, then $f(-x) = -f(x)$. Thus $g(-x) = f(-x) - 2 = -f(x) - 2$ whereas $-g(x) = -(f(x) - 2) = -f(x) + 2$. Then g cannot be odd because $g(-x) = -g(x) \Rightarrow -f(x) - 2 = -f(x) + 2$ $\Rightarrow 4 = 0$, which is a contradiction. Also, g(x) is not even unless $f(x) = 0$ for all x. On the other hand, if f is even, then $g(x) = f(x) - 2$ is also even: $g(-x) = f(-x) - 2 = f(x) - 2 = g(x)$.

16. If g is odd and g(0) is defined, then $g(0) = g(-0) = -g(0)$. Therefore, $2g(0) = 0 \Rightarrow g(0) = 0$.

17. For (x,y) in the 1st quadrant, $|x|+|y| = 1+x \Leftrightarrow x+y = 1+x \Leftrightarrow y = 1$. For (x,y) in the 2nd quadrant, $|x|+|y| = x+1 \Leftrightarrow -x+y = x+1 \Leftrightarrow y = 2x+1$. In the 3rd quadrant, $|x|+|y| = x+1 \Leftrightarrow -x-y = x+1 \Leftrightarrow y = -2x-1$. In the 4th quadrant, $|x|+|y| = x+1 \Leftrightarrow x+(-y) = x+1 \Leftrightarrow y = -1$. The graph is given at the right.



18. We use reasoning similar to Exercise 17.
    (1) 1st quadrant: $y+|y| = x+|x| \Leftrightarrow 2y = 2x \Leftrightarrow y = x$.
    (2) 2nd quadrant: $y+|y| = x+|x| \Leftrightarrow 2y = x+(-x) = 0 \Leftrightarrow y = 0$.
    (3) 3rd quadrant: $y+|y| = x+|x| \Leftrightarrow y+(-y) = x+(-x) \Leftrightarrow 0 = 0$ $\Rightarrow$ all points in the 3rd quadrant satisfy the equation.
    (4) 4th quadrant: $y+|y| = x+|x| \Leftrightarrow y+(-y) = 2x \Leftrightarrow 0 = x$. Combining these results we have the graph given at the right:



19. If f is even and odd, then $f(-x) = -f(x)$ and $f(-x) = f(x) \Rightarrow f(x) = -f(x)$ for all x in the domain of f. Thus $2f(x) = 0 \Rightarrow f(x) = 0$.

20. (a) As suggested, let $E(x) = \frac{f(x)+f(-x)}{2} \Rightarrow E(-x) = \frac{f(-x)+f(-(-x))}{2} = \frac{f(x)+f(-x)}{2} = E(x) \Rightarrow E$ is an even function. Define $O(x) = f(x) - E(x) = f(x) - \frac{f(x)+f(-x)}{2} = \frac{f(x)-f(-x)}{2}$. Then
    $O(-x) = \frac{f(-x)-f(-(-x))}{2} = \frac{f(-x)-f(x)}{2} = -\left(\frac{f(x)-f(-x)}{2}\right) = -O(x) \Rightarrow O$ is an odd function $\Rightarrow f(x) = E(x) + O(x)$ is the sum of an even and an odd function.
    (b) Part (a) shows that $f(x) = E(x) + O(x)$ is the sum of an even and an odd function. If also $f(x) = E_1(x) + O_1(x)$, where E_1 is even and O_1 is odd, then $f(x) - f(x) = 0 = (E_1(x) + O_1(x)) - (E(x) + O(x))$. Thus, $E(x) - E_1(x) = O_1(x) - O(x)$ for all x in the domain of f (which is the same as the domain of $E - E_1$ and $O - O_1$). Now $(E - E_1)(-x) = E(-x) - E_1(-x) = E(x) - E_1(x)$ (since E and E_1 are even) $= (E - E_1)(x) \Rightarrow E - E_1$ is even. Likewise, $(O_1 - O)(-x) = O_1(-x) - O(-x) = -O_1(x) - (-O(x))$ (since O and O_1 are odd) $= -(O_1(x) - O(x)) = -(O_1 - O)(x) \Rightarrow O_1 - O$ is odd. Therefore, $E - E_1$ and

$O_1 - O$ are both even and odd so they must be zero at each x in the domain of f by Exercise 19. That is, $E_1 = E$ and $O_1 = O$, so the decomposition of f found in part (a) is unique.

21. If the graph of f(x) passes the horizontal line test, so will the graph of $g(x) = -f(x)$ since it's the same graph reflected about the x-axis.
Alternate answer: If $g(x_1) = g(x_2)$ then $-f(x_1) = -f(x_2)$, $f(x_1) = f(x_2)$, and $x_1 = x_2$ since f is one-to-one.

22. Suppose that $g(x_1) = g(x_2)$. Then $\frac{1}{f(x_1)} = \frac{1}{f(x_2)}$, $f(x_1) = f(x_2)$, and $x_1 = x_2$ since f is one-to-one.

23. (a) The expression $a(b^{c-x}) + d$ is defined for all values of x, so the domain is $(-\infty, \infty)$. Since b^{c-x} attains all positive values, the range is (d, ∞) if $a > 0$ and the range is $(-\infty, d)$ if $a < 0$.
(b) The expression $a \log_b (x - c) + d$ is defined when $x - c > 0$, so the domain is (c, ∞).
Since $a \log_b (x - c) + d$ attains every real value for some value of x, the range is $(-\infty, \infty)$.

24. (a) Suppose $f(x_1) = f(x_2)$. Then:

$$\frac{ax_1 + b}{cx_1 + d} = \frac{ax_2 + b}{cx_2 + d}$$

$$(ax_1 + b)(cx_2 + d) = (ax_2 + b)(cx_1 + d)$$

$$acx_1x_2 + adx_1 + bcx_2 + bd = acx_1x_2 + adx_2 + bcx_1 + bd$$

$$adx_1 + bcx_2 = adx_2 + bcx_1$$

$$(ad - bc)x_1 = (ad - bc)x_2$$

Since $ad - bc \neq 0$, this means that $x_1 = x_2$.

(b) $y = \frac{ax + b}{cx + d}$

$$cxy + dy = ax + b$$

$$(cy - a)x = -dy + b$$

$$x = \frac{-dy + b}{cy - a}$$

Interchange x and y.

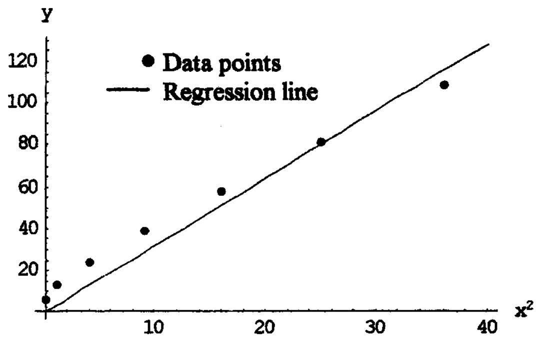
$$y = \frac{-dx + b}{cx - a}$$

$$f^{-1}(x) = \frac{-dx + b}{cx - a}$$

(c) As $x \to \pm\infty$, $f(x) = \frac{ax + b}{cx + d} \to \frac{a}{c}$, so the horizontal asymptote is $y = \frac{a}{c}$ $(c \neq 0)$. Since f(x) is undefined at $x = -\frac{d}{c}$, the vertical asymptote is $x = -\frac{d}{c}$ provided $c \neq 0$.

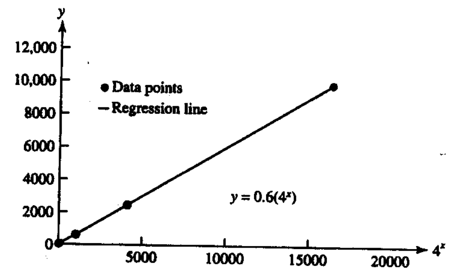
(d) As $x \to \pm\infty$, $f^{-1}(x) = \frac{-dx + b}{cx - a} \to -\frac{d}{c}$, so the horizontal asymptote is $y = -\frac{d}{c}$ $(c \neq 0)$. Since $f^{-1}(x)$ is undefined at $x = \frac{a}{c}$, the vertical asymptote is $x = \frac{a}{c}$. The horizontal asymptote of f becomes the vertical asymptote of f^{-1} and vice versa due to the reflection of the graph about the line $y = x$.

25. (a)



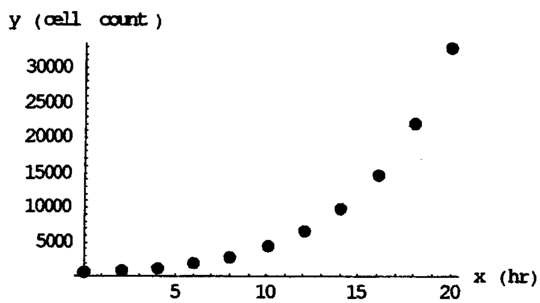
The graph does not support the assumption that $y \propto x^2$

(b)

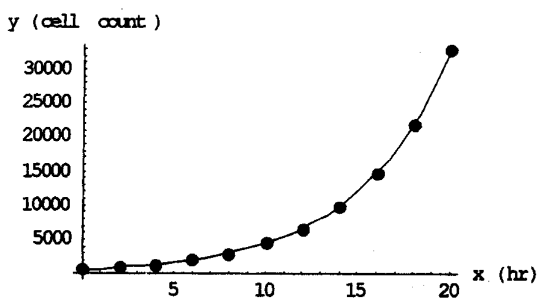


The graph supports the assumption that $y \propto 4^x$. The constant of proportionality is estimated from the slope of the regression line, which is 0.6, therefore, $y = 0.6\left(4^x\right)$.

26. Plot the data.



The graph suggests that an exponential relationship might be appropriate. The exponential regression function on the TI-92 Plus calculator gives $y = 599e^{0.2x}$ and the following graph shows the exponential regression curve superimposed on the graph of the data points.

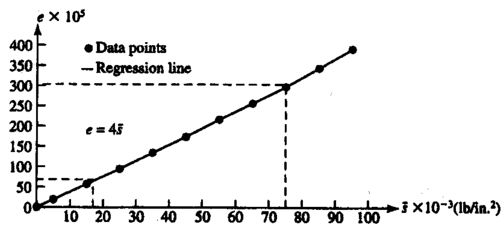


The curve appears to fit the data very well.

The cell count reaches 50,000 when $50{,}000 = 599e^{0.2x} \Rightarrow x = 5 \ln \dfrac{50{,}000}{599} \approx 22.123$ hours ≈ 22 hours 7.4 minutes.

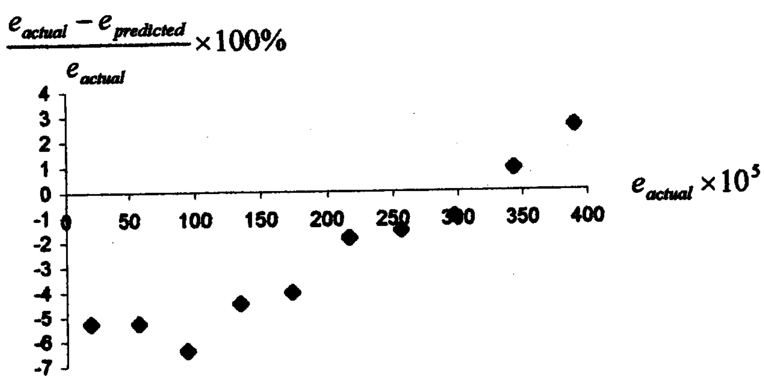
27. (a) Since the elongation of the spring is zero when the stress is $5(10^{-3})(\text{lb/in.}^2)$, the data should be adjusted by subtracting this amount from each of the stress data values. This gives the following table, where $\bar{s} = s - 5(10^{-3})$.

$\bar{s} \times 10^{-3}$	0	5	15	25	35	45	55	65	75	85	95
$e \times 10^5$	0	19	57	94	134	173	216	256	297	343	390



The slope of the graph is $\dfrac{(297-57)(10^5)}{(75-15)(10^{-3})} = 4.00(10^8)$ and the model is $e = 4(10^8)\bar{s}$ or $e = 4(10^8)(s - 5(10^{-3}))$.

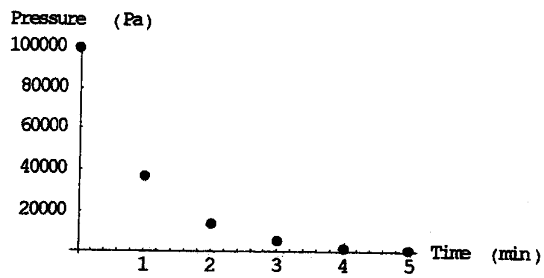
(b) As show in the following graph, the largest relative error is about 6.4%



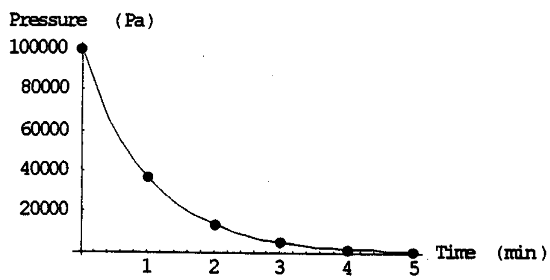
The model fits the data well. There does appear to be a pattern in the errors (i.e., they are not random) indicating that a refinement of the model is possible.

(c) $e = 4(10^8)(200-5)(10^{-3}) = 780(10^5)(\text{in./in.})$. Since $s = 200(10^{-3})(\text{lb/in.}^2)$ is well outside the range of the data used for the model, one should not feel comfortable with this prediction without further testing of the spring.

28. Plot the data.



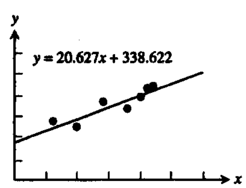
(a) The data suggests a decaying exponential relationship. The exponential regression function on the TI-92 Plus calculator gives $p = 100{,}085e^{-t}$ where p is the pressure in pascals and t is the elapsed time in minutes. The next graph superimposes the exponential regression curve on the data points.



(b) The graph shows that the exponential regression fits the data very well.

(c) The pressure reaches 200 Pa when $200 = 100{,}085e^{-t} \Rightarrow t = -\ln\left(\frac{200}{100{,}085}\right) \approx 6.22$ minutes ≈ 6 minutes 13 seconds.

29. (a) $y = 20.627x + 338.622$



(b) When $x = 30$, $y = 957.445$. According to the regression equation, about 957 degrees will be earned.

(c) The slope is 20.627. It represents the approximate annual increase in the number of doctorates earned by Hispanic Americans per year.

30. (a) $y = 14.60175 \cdot 1.00232^x$

(b) Solving $y = 25$ graphically, we obtain $x \approx 232$. According to the regression equation, the population will reach 25 million in the year 2132.

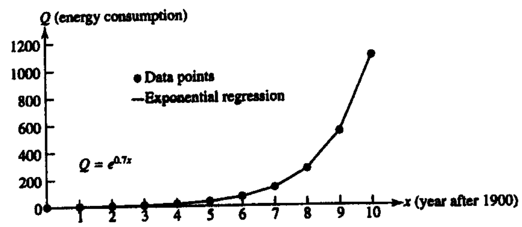
(c) 0.232%

31. (a) The TI-92 Plus calculator gives $f(x) = 2.000268 \sin(2.999187x - 1.000966) + 3.999881$.

(b) $f(x) = 2 \sin(3x - 1) + 4$

32. (a) $y = -590.969 + 152.817 \ln x$, where x is the number of years after 1960.
(b) When $x = 85$, $y \approx 87.94$.
About 87.94 million metric tons were produced.
(c)
$$\begin{aligned} -590.969 + 152.817 \ln x &= 120 \\ 152.817 \ln x &= 710.969 \\ \ln x &= \frac{710.969}{152.817} \\ x &= e^{\frac{710.969}{152.817}} \approx 104.84 \end{aligned}$$
According to the regression equation, oil production will reach 120 million metric tons when $x \approx 104.84$, in about 2005.

33. (a) The TI-93 Plus calculator gives $Q = 1.00(2.0138^x) = 1.00e^{0.7x}$



(b) For 1996, $x = 9.6 \Rightarrow Q(9.6) = e^{0.7(9.6)} = 828.82$ units of energy consumed that year as estimated by the exponential regression. The exponential regression shows that energy consumption has doubled (i.e., increased by 100%) each decade during the 20th century. The annual rate of increase during this time is $e^{0.7(0.1)} - e^{0.7(0)} = 0.0725 = 7.25\%$.

NOTES: