



Case 3 - Defect Sampling: Sampling Distributions and Sampling Error

Defect Sampling: Sampling Distributions and Sample Error

Background

Defect sampling is used in industrial settings to determine the types and amounts of defects in manufactured items. Items at various stages of production are removed from the process and inspected for defects. Sustained testing allows operations managers to discover whether some part of the manufacturing process is failing to meet performance criteria and product standards. To minimize manufacturing defects, early detection and problem resolution are critical.

ACME Manufacturing produces electronic components used in the assembly of industrial robotic equipment. The components are technologically sophisticated, and quality must be rigorously monitored to ensure that specifications are met. Of course, inspection can be a time consuming and costly activity. ACME production managers must determine how much sampling and testing is needed to minimize costs and maximize yields.

In the current sampling plan, one component from the production line is randomly selected every 15 minutes. Each component is inspected and tested for major and minor defects. Major defects, which affect component performance, must be addressed immediately. Fortunately, major defects are rare and are generally contained and corrected early in the process. Minor defects, such as nicks and scratches, are those that affect the appearance of a component but not its functionality. Monitoring minor defects to ensure the rate doesn't increase is of particular concern to ACME management.

The Task

ACME currently inspects one part every 15 minutes during an eight hour shift. Production managers have complained this sampling rate is too high, which has driven up costs and decreased output. They suggest implementing sampling plans involving fewer parts.

In this case, we explore the effectiveness of the current defect sampling plan, and several alternative plans, in detecting changes in the occurrence of minor defects.

The Data [Defects.jmp](#)

The data set contains ten days of data on minor defects. Each day, one item is tested every fifteen minutes during an eight-hour shift. The variables in the data set are:

| | |
|----------------|---|
| Day | Day of the test: 1 – 10 |
| Sample | Time of the day that sample was taken in military time (e.g., 13:00 is 1pm) |
| Defects | Number of minor defects detected on the sampled item |

Analysis

A visual examination of the data set is a good place to start. Exhibit 1 organizes the 15-minute data by day. The first thing we note is that the production process is far from error-free. That's not too surprising, since the errors recorded here are minor ones.

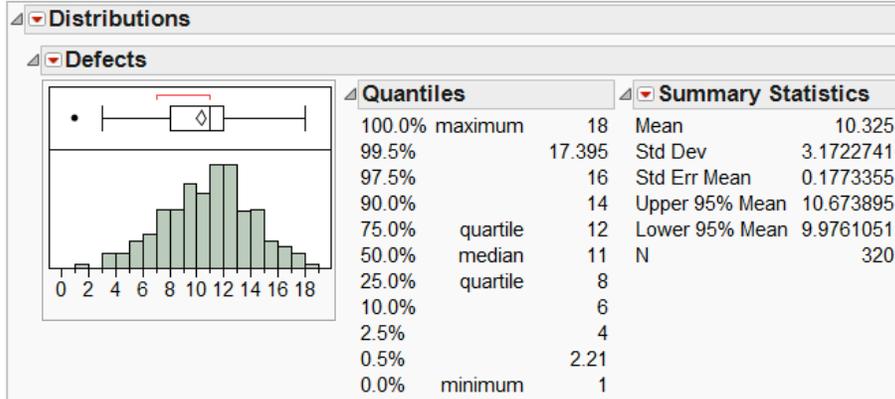
Exhibit 1 Defects Split by Day

| Sample | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|----------|----|----|----|----|----|----|----|----|----|----|
| 1 08:15 | 12 | 17 | 7 | 11 | 7 | 13 | 10 | 10 | 11 | 9 |
| 2 08:30 | 8 | 12 | 11 | 11 | 8 | 9 | 8 | 11 | 12 | 18 |
| 3 08:45 | 9 | 7 | 9 | 8 | 7 | 14 | 15 | 11 | 10 | 17 |
| 4 09:00 | 11 | 11 | 16 | 12 | 11 | 10 | 3 | 11 | 12 | 16 |
| 5 09:15 | 9 | 7 | 8 | 12 | 7 | 7 | 8 | 14 | 14 | 10 |
| 6 09:30 | 10 | 6 | 12 | 11 | 17 | 7 | 12 | 12 | 13 | 12 |
| 7 09:45 | 12 | 10 | 14 | 12 | 7 | 11 | 5 | 15 | 6 | 8 |
| 8 10:00 | 9 | 11 | 5 | 13 | 8 | 12 | 5 | 10 | 8 | 9 |
| 9 10:15 | 12 | 12 | 13 | 10 | 11 | 9 | 14 | 6 | 9 | 11 |
| 10 10:30 | 4 | 7 | 14 | 15 | 3 | 11 | 12 | 13 | 13 | 13 |
| 11 10:45 | 11 | 11 | 6 | 12 | 14 | 13 | 16 | 14 | 5 | 11 |
| 12 11:00 | 8 | 6 | 4 | 6 | 7 | 11 | 9 | 10 | 11 | 8 |
| 13 11:15 | 12 | 12 | 14 | 7 | 10 | 12 | 10 | 11 | 6 | 9 |
| 14 11:30 | 12 | 13 | 8 | 5 | 3 | 6 | 10 | 10 | 8 | 14 |
| 15 11:45 | 9 | 1 | 11 | 3 | 11 | 14 | 12 | 9 | 11 | 9 |
| 16 12:00 | 8 | 12 | 10 | 13 | 14 | 12 | 4 | 9 | 14 | 4 |
| 17 12:15 | 9 | 13 | 6 | 9 | 10 | 10 | 13 | 12 | 11 | 12 |
| 18 12:30 | 10 | 8 | 10 | 11 | 13 | 9 | 9 | 12 | 12 | 13 |
| 19 12:45 | 15 | 14 | 5 | 9 | 16 | 9 | 11 | 3 | 16 | 11 |
| 20 13:00 | 11 | 12 | 9 | 15 | 12 | 5 | 12 | 10 | 11 | 9 |
| 21 13:15 | 14 | 8 | 12 | 12 | 10 | 11 | 12 | 9 | 7 | 10 |
| 22 13:30 | 11 | 12 | 12 | 6 | 11 | 17 | 7 | 9 | 5 | 8 |
| 23 13:45 | 9 | 12 | 10 | 8 | 11 | 13 | 8 | 9 | 16 | 10 |
| 24 14:00 | 7 | 10 | 13 | 11 | 13 | 10 | 15 | 5 | 7 | 9 |
| 25 14:15 | 15 | 10 | 4 | 14 | 9 | 7 | 11 | 11 | 11 | 14 |
| 26 14:30 | 11 | 15 | 6 | 7 | 9 | 5 | 14 | 8 | 5 | 11 |
| 27 14:45 | 13 | 6 | 13 | 7 | 9 | 12 | 13 | 14 | 8 | 8 |
| 28 15:00 | 9 | 12 | 9 | 16 | 13 | 6 | 7 | 17 | 14 | 15 |
| 29 15:15 | 15 | 13 | 17 | 3 | 12 | 12 | 10 | 7 | 10 | 10 |
| 30 15:30 | 8 | 7 | 15 | 13 | 14 | 4 | 16 | 14 | 9 | 8 |
| 31 15:45 | 7 | 7 | 14 | 16 | 14 | 10 | 10 | 12 | 11 | 9 |
| 32 16:00 | 11 | 11 | 12 | 9 | 8 | 14 | 7 | 6 | 12 | 13 |

(Tables > Split; Use Defects as the Split Column and Day as Split By. Under Remaining Columns select Keep All)

From Exhibit 2, we see that the average number of defects per item is 10.325 and the standard deviation is 3.17.

Exhibit 2 Distribution of **Defects**



(Return to the original table. Analyze > Distribution; Use Defects as Y, Columns.)

Over this ten-day period, 320 items were sampled and tested. That's 32 items per day! A summary report of these tests shows that just over ten defects are detected on average each day (Exhibit 3).

Exhibit 3 Summary of **Defects** by **Day**

| Day | Defects | |
|-----|---------|-------|
| | N | Mean |
| 1 | 32 | 10.34 |
| 2 | 32 | 10.16 |
| 3 | 32 | 10.28 |
| 4 | 32 | 10.22 |
| 5 | 32 | 10.28 |
| 6 | 32 | 10.16 |
| 7 | 32 | 10.25 |
| 8 | 32 | 10.44 |
| 9 | 32 | 10.25 |
| 10 | 32 | 10.88 |

(Tables > Tabulate; drag Day in drop zone for rows as a grouping column, and Defects in the drop zone for column as an analysis column. Then, drag N and Mean from the middle panel to the middle of the table.)

Let's consider two alternative sampling schemes.

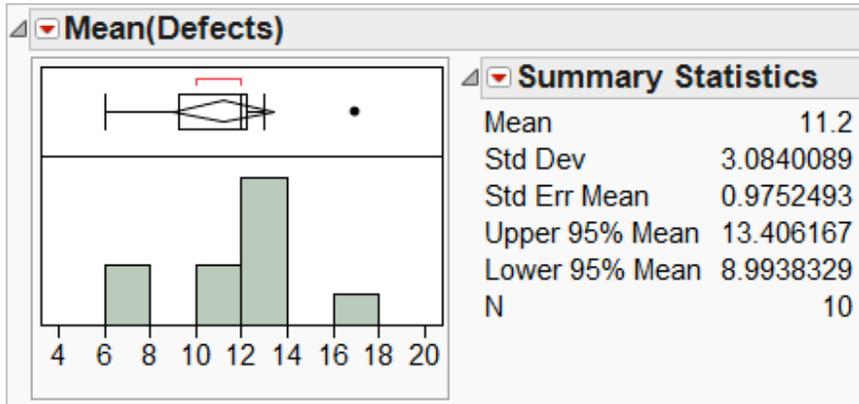
Suppose that instead of sampling every fifteen minutes, we had only sampled once each day, say at 9:30am; we refer to this as Sampling Scheme A. This cuts down the number of samples each day from 32 to 1, presumably a significant decrease in time and money.

Alternatively, we could have sampled twice per day, say at 9:30am and 2:30pm (14:30) and averaged the morning and afternoon readings. We'll call this Sampling Scheme B. Exhibit 4 compares the results of Schemes A and B.

Exhibit 4 Sampling Schemes A and B

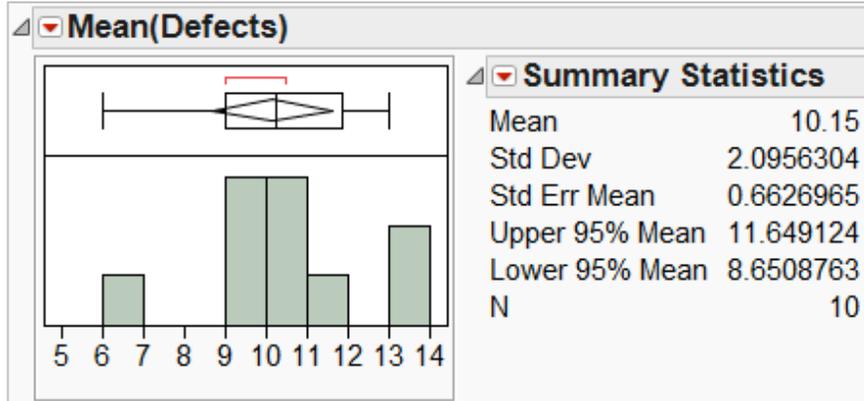
Sampling Scheme A: Sample Once Per Day (9:30am)

| Day | Defects | |
|-----|---------|------|
| | N | Mean |
| 1 | 1 | 10 |
| 2 | 1 | 6 |
| 3 | 1 | 12 |
| 4 | 1 | 11 |
| 5 | 1 | 17 |
| 6 | 1 | 7 |
| 7 | 1 | 12 |
| 8 | 1 | 12 |
| 9 | 1 | 13 |
| 10 | 1 | 12 |



Sampling Scheme B: Sample Twice per Day (9:30 and 14:30)

| Day | Defects | |
|-----|---------|------|
| | N | Mean |
| 1 | 2 | 11 |
| 2 | 2 | 9 |
| 3 | 2 | 12.5 |
| 4 | 2 | 10.5 |
| 5 | 2 | 14 |
| 6 | 2 | 8 |
| 7 | 2 | 13 |
| 8 | 2 | 9 |
| 9 | 2 | 11 |
| 10 | 2 | 11.5 |



(Rows > Data Filter; select **Sample** and click Add. Then, select 9:30 and check the Show and Include boxes. Use the control key to add 14:30 to the selection. Use Tables > Tabulate to create the summary table as described earlier. Under the top red triangle in Tabulate, select "Make Into Data Table". Then, use Analyze > Distribution to create a distribution of Mean(Defects).)

In Exhibit 4 we note that the Scheme A generated *individual* observations (i.e., only one sample was taken each day) with a mean of 11.2 defects and a standard deviation of 3.084. Scheme B, whose observations are *averages* of two samples per day, generates an overall average of 10.15 defects with a standard deviation of 2.096. Note the drop in the standard deviation, from 3 to 2, a decrease of one-third. The first important result of this case has emerged:

Averages vary less than individual observations.

This result is intuitively appealing since, by their nature, averages "average out" extremes, and, therefore, vary less.

The histograms in Exhibit 5 show the sample averages for the five different sampling schemes (1, 2, 8, 16 and 32 samples per day). Pay close attention to what happens to the *spread* of the averages in the histograms as we collect an increasingly larger number of samples each day. In particular, note that the sample size, n , increases as you move from Scheme A to Scheme E while at the same time the *variation* in the histogram decreases.

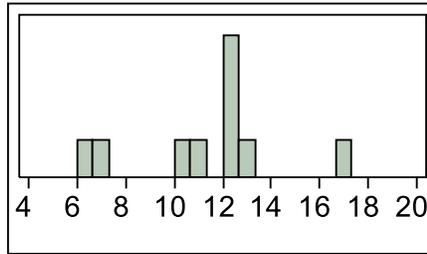
Exhibit 5 Five Sampling Schemes

Sampling Scheme A

Sample at 9:30

Sample size each day:

$n = 1$

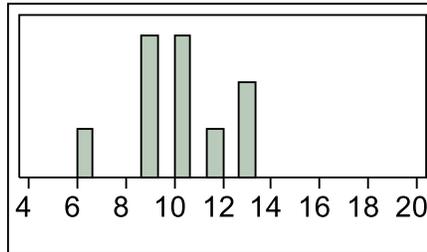


Sampling Scheme B

Sample at 9:30 and 14:30

Sample size each day:

$n = 2$

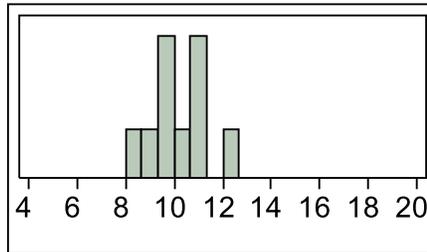


Sampling Scheme C

Sample every hour beginning at 8:30

Sample size each day:

$n = 8$

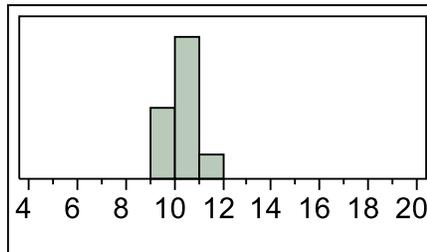


Sampling Scheme D

Sample every half-hour beginning at 8:30

Sample size each day:

$n = 16$



Sampling Scheme E

Sample every 15 minutes

Sample size each day:

$n = 32$

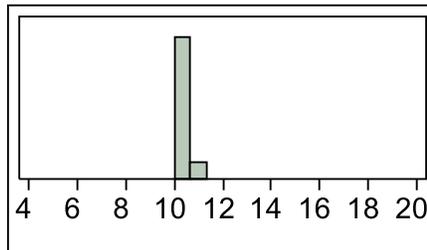


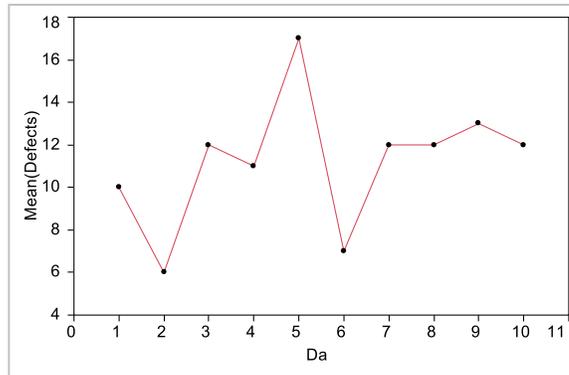
Exhibit 5 illustrates the second important lesson learned from this case:

The *variability* in averages decreases as the sample size increases.

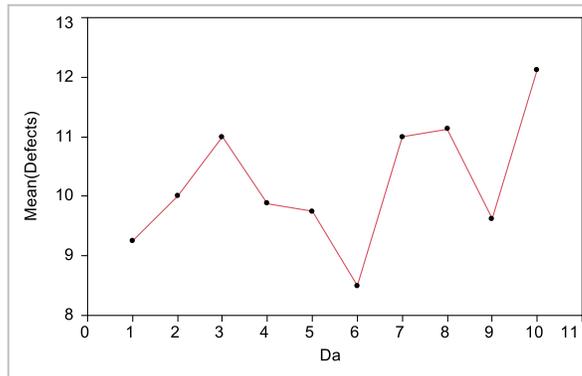
Now return to Exhibit 3 and note that there appears to have been a subtle shift in the process on Day 10. The average of Day 10 is larger than the averages on the other days. Exhibit 6 shows daily time series plots of Sampling Schemes A, C and E. Note the differences in the time series plots with regard to their ability to detect the shift in the process on Day 10.

Exhibit 6 Average Defects Plotted Over Time

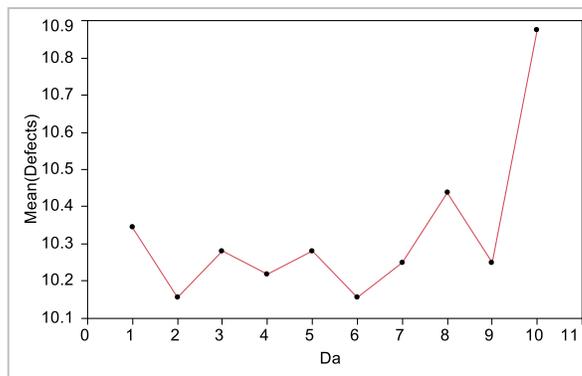
Sampling Scheme A
 Sample at 9:30
 Sample size each day:
 $n = 1$



Sampling Scheme C
 Sample every hour
 beginning at 8:30
 Sample size each day:
 $n = 8$



Sampling Scheme E
 Sample every 15
 minutes
 Sample size each day:
 $n = 32$



(Use Data Filter and Tabulate, as described earlier, to create the summarized data tables for each scheme. Then, Graph > Overlay Plot; Use Mean(Defects) as Y and Day as X, and click OK. In the Overlay Plot window, select "Connect Through Missing" under the red triangle.)

In Exhibit 6, the upward shift in the process on Day 10 goes undetected in Sampling Scheme A. That's because some, but not all, readings taken on Day 10 shifted upward. Sampling Scheme A, with its single sample each day, has a smaller chance of sampling an item that displays the shift in the process. Sampling Schemes C and E, with increasingly larger sample sizes each day, more readily detect the shift on Day 10. By sampling throughout the day, Schemes C and E provide a more representative estimate of performance on that day, and:

- Averages are comprised of multiple individual observations, some of which will likely be the larger (or smaller) values if the process has shifted, and
- Averages are not resistant. If a sample contains outliers (in this case, large outliers), the average will move in the direction of the outlier.

From Exhibit 6 we learn that:

Averages are more likely to detect changes in a process than are individual observations. Up to a point, the larger the sample size the more likely a shift in the process will be discovered.

Of course, many different sampling plans can be considered – we've explored only a few here. The challenge is to achieve a balance between the cost of inspection and the impact of undetected defects.

Summary

Statistical Insights

With this case, we see that sample statistics, such as averages, are variable. That's because different samples drawn from a fixed population will generate different sample averages. For example, when sampling every fifteen minutes (Exhibit 3), the first daily average was 10.34. The second day's average declined to 10.16. By Day 3, the average was 10.28. *The averages varied day to day, depending on the particular sample items chosen.* We also learned that:

- When sampling from a fixed population, individual observations vary more than averages.
- The *variability* in averages *decreases* as the sample size *increases*.
- Larger sample sizes are more likely to detect shifts in a process than smaller sample sizes.

The ability to detect a shift also depends on the size or magnitude of the change. Detecting larger shifts requires less data than detecting smaller shifts.

Note that our focus in this case was sampling distributions and sampling error. A more formal tool for monitoring a process over time is a control chart, which is a type of time series plot used to plot either individual values or averages over time. Other statistical tools, such as hypothesis tests and capability studies, might also be used to analyze minor defect levels.

Managerial Implications

In defect sampling, the decision concerning the number of items to sample has important implications on both cost and utility:

- If a single observation is sampled each day, costs are low. However, expect a lot of variability in the samples from day to day and keep in mind that the data tend to be slow in detecting shifts in the process.
- Multiple samples each day, whose values are then averaged to create that day's reading, are more likely to detect shifts in the process than a single daily reading. However, the costs of sampling will increase.

Management must weigh the costs of sampling versus the potential cost and consequences of undetected process changes.

JMP Features and Hints

In this case, we:

- Used Split to restructure our data into multiple columns,
- Used the Data Filter to select specific values in the data table and hide and exclude other values,
- Used Tabulate and Distribution to summarize our data, and
- Used Overlay Plot to create time series plots.

Exercises

Open the **Sampling Distribution of Sample Means** teaching module in JMP. The module can be downloaded from: http://www.jmp.com/academic/learning_modules.shtml.

The screenshot displays the JMP Pro interface for the 'Sampling Distribution of Sample Means v1.5' teaching module. The main window is divided into several panels:

- Population Characteristics:** Population Shape: Normal; Population Mean: 0; Population Standard Deviation: 1; Name of Variable: Height.
- Demo Characteristics:** Sample Size: 25; Number of Samples: 1; Animate Illustration? (No selected).
- Run Simulation:** Draw Samples, Reset Samples buttons.
- JMP Data Table:** Create Data Table button.
- Population Data:** Create Histogram button.
- Distribution of Sample Data:** A plot showing a normal distribution curve for 'Height' with x-axis from -4 to 4 and y-axis from 0 to 20.
- Normal Quantile Plot:** A normal quantile plot for 'Height'.
- Distribution of Sample Means:** A plot showing a normal distribution curve for 'Mean Height' with x-axis from -4 to 4 and y-axis from 0 to 10.
- Sample Summary Table:** A table with columns: Mean, Std Dev, Sample.
- Means Summary Table:** A table with columns: Mean of Sample Means, Std Dev of Sample Means, No. of Sample Means.

Buttons at the bottom include Help, More Information, and Restart Demo. The JMP logo is in the bottom left corner.

This module will randomly draw a number of samples from a distribution. By default, samples of size 25 will be drawn from the standard normal distribution (the mean is 0 and the standard deviation is 1.)

1. Change the Sample Size to 5, and change the Number of Samples to 1000. Click Draw Samples. The Distribution of Sample Means will display a histogram for the 1000 sample means.
 - a. What is the shape of the histogram? What is the center of the histogram? How spread out is the histogram?
 - b. The Means Summary Table will display the Mean of the Sample Means and the Std Dev of the Sample Means. Record this information.
2. Change the Sample Size to 100, and click Draw Samples.
 - a. Look at the Distribution of Sample Means. How does the shape, center and spread of this histogram compare to the previous histogram?
 - b. Record the Mean of the Sample Means and the Std Dev of the Sample Means. How do these values compare to the previous values? Are these results consistent with what we have learned in this case study?
3. Finally, change the Population Shape to Skewed Right, change the Population Standard Deviation to 5, and click Draw Samples.
 - a. Samples are being drawn from a population that is highly right skewed (note that the distribution of the last sample drawn under Distribution of Sample Data is right skewed). Look at the Distribution of Sample Means. Describe the shape, center and spread of this histogram.
 - b. Record the Mean of the Sample Means. How does this value compare to the Population Mean?



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