Preliminaries

P.1 LINES

OBJECTIVE A: Find the net changes Δx and Δy in a particle's coordinates as it moves from a point P to a point Q.

1. If a particles starts at $P(2, -1)$ and goes to $Q(-7, -3)$, then its x-coordinate changes by

$$\Delta x = -7 - \_\_\_\_\_\_\_\_ = \_\_\_\_\_\_\_\_.$$

2. Its y-coordinate changes by

$$\Delta y = \_\_\_\_\_\_\_\_ - (-1) = \_\_\_\_\_\_\_\_.$$

OBJECTIVE B: Given the increments from the point P to the point Q and the coordinates of one of these points, determine the coordinates of the other point.

3. The coordinates of a particle change by $\Delta x = -3$ and $\Delta y = 5$ in moving from $P(1, -4)$ to $Q(x, y)$. The x-coordinate of Q is given by

$$x = 1 + \_\_\_\_\_\_\_\_ = \_\_\_\_\_\_\_\_.$$

4. The y-coordinate of Q is given by

$$y = \_\_\_\_\_\_\_\_ + 5 = \_\_\_\_\_\_\_\_.$$

OBJECTIVE C: Define the *slope* of a straight line and calculate the slope (if any) of the line determined by two given points.

5. The *slope* of the line through the points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ is given by

$$m = \frac{\text{rise}}{\text{run}} = \_\_\_\_\_\_\_\_ = \_\_\_\_\_\_\_\_, \text{ provided that } x_1 \neq x_2.$$

6. If $x_1 = x_2$, then the line throught the points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ is a ____________ line. For vertical lines, the ________ is not defined.

7. The slope of the line through the points $A\left(-\frac{1}{2}, 1\right)$, $B(0, -2)$ is $m =$ ________.

OBJECTIVE D: Find the slope (if any) of a line perpendicular to a line determined by two given points.

8. The slope of the line perpendicular to AB in Problem 7 is $m =$ ________.

OBJECTIVE E: Write an equation of any vertical line given a point on the line.

9. An equation of the vertical line passing through the point $P(4, -7)$ is ________.

OBJECTIVE F: Write an equation of any line with given slope and passing through a given point.

10. Using the *point-slope* equation of the line, we have $y = y_1 + m(x - x_1)$. Thus an equation of the line with slope $m = -2$ through the point $(1, 3)$ is given by ________________.

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1. 2, –9

2. –3, –2

3. –3, –2

4. –4, 1

5. $\dfrac{y_2 - y_1}{x_2 - x_1}, \dfrac{y_1 - y_2}{x_1 - x_2}$

6. vertical, slope

7. –6

8. $\dfrac{1}{6}$

9. $x = 4$

10. $y = 3 + (-2)(x - 1)$ or $y = 5 - 2x$

11. The line perpendicular to the line in Problem 10 has slope $m = -\left(\frac{1}{-2}\right) = \frac{1}{2}$. Thus an equation of the perpendicular through (1, 3) is ______________________.

OBJECTIVE G: Write an equation of any line given two points on the line.

12. Let $P_1(-3, 0)$ and $P_2(2, -1)$ be two points on the line L. The slope of L is $m =$ ____________. Thus, an equation of L using P_1 is ____________________, using P_2 an equation is ____________________. In either case, solving for y we obtain the equation $y =$ ____________________.

13. Let $P_1(1, -3)$ and $P_2(1, 5)$ be two points on the line L. Since the x-coordinates of the points are the same, we conclude that L is a ________________ line and hence has no ________________. An equation for L is ____________________.

OBJECTIVE H: Recognize an equation as representing a line and determine the slope (if any), the x-intercept (if any), and the y-intercept (if any).

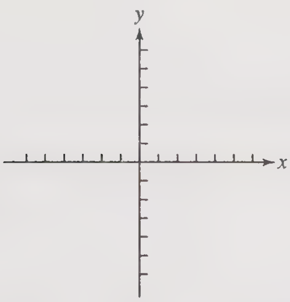
14. The equation $3x - 2y = 6$ represents a straight line because it contains only ________________ powers of x and y. When $x = 0$, $y =$ ____________, which gives the value where the line crosses the y-axis. This is called the ________________. When $y = 0$, $x =$ ____________, giving the value where the line crosses the ____________. This is called the x-intercept.

15. The equation $y = 3$ represents a straight line that is parallel to the ________________. It is called a ____________ line and has slope $m =$ ________________.

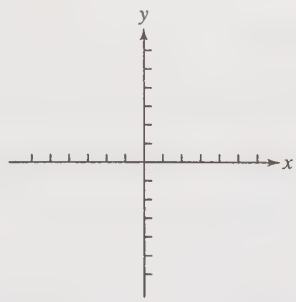
16. The equation $xy = 1$ does not represent a straight line because it is not a ____________________ equation when the variables x and y are multiplied together.

OBJECTIVE I: Graph any equation representing a line.

17. Graph the line $y = -3x + 1$.



18. Graph the line $\frac{x}{2} - \frac{y}{3} = \frac{1}{2}$.



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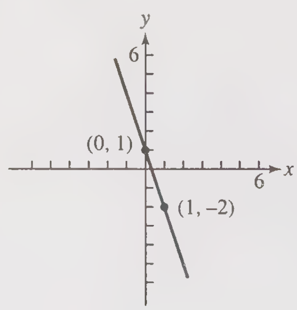
11. $y = 3 + \frac{1}{2}(x - 1)$ or $y = \frac{5}{2} + \frac{1}{2}x$ **12.** $-\frac{1}{5}$, $y = 0 + \left(-\frac{1}{5}\right)(x + 3)$, $y = -1 + \left(-\frac{1}{5}\right)(x - 2)$, $-\frac{x+3}{5}$

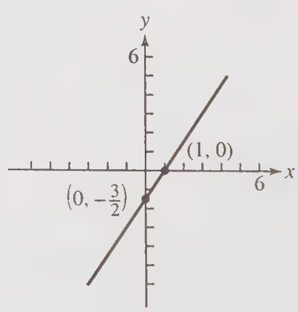
13. vertical, slope, $x = 1$ **14.** first, –3, y-intercept, 2, x-axis **15.** x-axis, horizontal, 0

16. linear

17.



18.



OBJECTIVE J: Find an equation of the line passing through a given point and parallel or perpendicular to a given line.

19. The line containing the point (–1, 2) that is parallel to the line $3x - y - 1 = 0$ has slope $m =$ __________. Since the line contains the point (–1, 2), its equation in point-slope form is __________.

20. The line containing the point (4, 1) that is perpendicular to the line $2y - 3x = 5$ has slope $m =$ __________. Since the line contains the point (4, 1), its equation in point-slope form is __________.

P.2 FUNCTIONS AND GRAPHS

In this section of the textbook, there are several terms associated with the concept of a function with which you will need to become familiar. The following items are designed to assist you in learning the precise mathematical meanings of these various terms.

21. Calculus is concerned with how variables are related. If to each value of the variable x there corresponds a unique value of the variable y, then y is said to be a __________ of x. The key word in this definition of function is __________: we do not want to input a single value for the variable x with two or more possible outcomes for y. Every function is determined by two things: (1) the __________ of the first variable x and (2) the __________ or condition describing how y is obtained from x. The variable x is called the __________ variable of the function; the second variable y is called the __________ variable. The set of values taken on by the dependent variable y is called the __________ of the function.

22. Two important restrictions on the domain of a real-valued function are: (a) never divide __________; (b) never take square roots of __________.

23. *Convention*: If the domain of a function is not stated explicitly, then the domain is automatically __________ for which the formula for the function gives __________. This is the function's __________ domain.

24. The *graph* of the function $y = f(x)$ is the set of points __________ whose coordinates are the __________ pairs of the function.

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19. 3, $y = 2 + 3(x + 1)$

20. $-\frac{2}{3}$, $y = 1 - \frac{2}{3}(x - 4)$

21. function, unique, domain, rule, independent, dependent, range

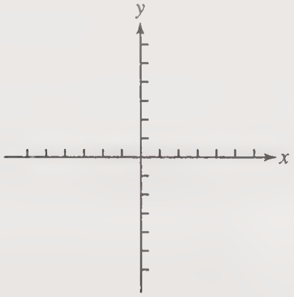
22. by zero, negative numbers

23. the largest set of x-values, real y-values, natural

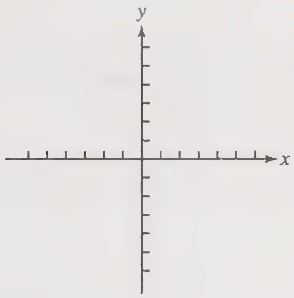
24. (x, y) in the plane, input-output

OBJECTIVE A: Given an equation for a function $y = f(x)$, calculate the value of f at a specified point, find the domain and range of f, and graph f by making a table of pairs.

25. Consider the function $y = -3x + 2$. The domain of the function is the interval ________________. Solving the equation for x, gives $x =$ ________________ so that the variable y may take on any value whatsoever. Thus, the range of the function is the interval ___________________. Sketch the graph.



26. Consider the function $y = \dfrac{x^2 - 1}{x + 1}$. The function is defined for all values of x except ________________; hence the domain consists of the union of the intervals ________________ and ________________. When $x \neq -1$, $y = \dfrac{x^2 - 1}{x + 1} = \dfrac{(x-1)(x+1)}{x+1} =$ ___________________. Therefore, the range of the function is all real numbers except for $y =$ ________________ (because $x \neq -1$), so that the range is the union of the two intervals __________ and ____________.



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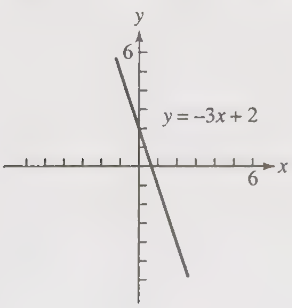
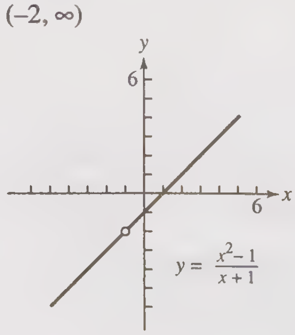
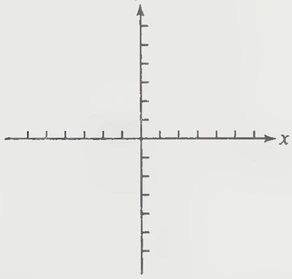
25. $(-\infty, \infty)$, $x = \dfrac{2 - y}{3}$, $(-\infty, \infty)$



26. $x = -1$, $(-\infty, -1)$, $(-1, \infty)$, $x - 1$, -2, $(-\infty, -2)$, $(-2, \infty)$



27. The domain of the function $y = -\sqrt{1-x}$ is the interval ______________, since $\sqrt{1-x}$ is defined whenever $1 - x \geq 0$. Squaring both sides and solving the resultant equation for x, we obtain $x =$ ______________. We see from this last equation that y can take on any value. However, since y is the negative square root, the range is the interval ______________.



28. If $g(x) = \dfrac{1}{\sqrt{x-2}}$, the domain of g is ______________. The value $g(3)$ is ______________; $g(11)$ is ______________; $g(a)$ is ______________; $g(b+2)$ is ______________.

29. Consider the function $y = [x-1] + 2$, where $[x-1]$ denotes the greatest integer in ______________. The domain of this function is the interval ______________. A table of some of the values for this function is given by (complete the table):

x	–2.0	–1.5	–1.0	–0.5	0	0.5	1.0	1.5	2.0	2.5
y	–1.0									

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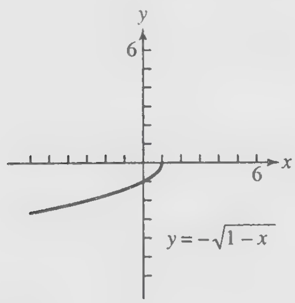
27. $(-\infty, 1]$, $x = 1 - y^2$, $(-\infty, 0]$

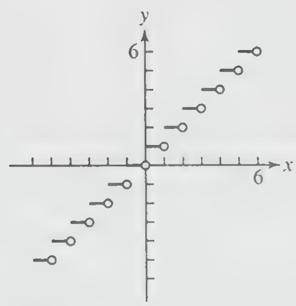


28. $(2, \infty)$, 1, $\dfrac{1}{3}$, $\dfrac{1}{\sqrt{a-2}}$, $\dfrac{1}{\sqrt{b}}$

29. $x - 1$, $(-\infty, \infty)$,

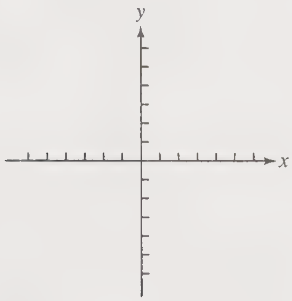
x	–2.0	–1.5	–1.0	–0.5	0
y	–1.0	–1.0	0	0	1.0

x	0.5	1.0	1.5	2.0	2.5
y	1.0	2.0	2.0	3.0	3.0



range: $\{\ldots, -2, -1, 0, 1, 2, 3, \ldots\}$

Sketch the graph of the function using the table. The range of this function is not an interval, but the set of numbers ______________________.



OBJECTIVE B: Test a given function to find what symmetries its graph has.

30. If whenever the point (x, y) lies on the graph, then the point $(-x, -y)$ also lies on the graph, we say that the graph is symmetric about the ______________.

31. If whenever the point (x, y) lies on the graph, then the point ______________ also lies on the graph, we say that the graph is symmetric about the x-axis.

32. If whenever the point (x, y) lies on the graph, then the point $(-x, y)$ also lies on the graph, we say that the graph is symmetric about the ______________.

33. The graph of the function $y = 3 - x^2$ is symmetric about the ________________. The reason is that (x, y) on the graph implies

$$y = 3 - x^2 \Rightarrow y = 3 - (-x)^2.$$

Therefore, the point ______________ also lies on the graph. The graph has no other symmetries.

OBJECTIVE C: For a given function $y = f(x)$, identify the function as even, odd, or neither.

34. The function $y = f(x)$ is even if $f(-x) =$ ______________. The graph of an even function is symmetric about ______________.

35. If $f(-x) = -f(x)$, the function $y = f(x)$ is said to be an ______________ function. The graph of an odd function is symmetric about the ________________.

36. For the function $f(x) = x^2 + x^4$, we find $f(-x) = (-x)^2 + (-x)^4 =$ ________________. Thus the function is ______________.

37. For the function $f(x) = x^2 - x^5$, we find $f(-x) = (-x)^2 - (-x)^5 =$ ________________. Is $f(-x) = f(x)$? Is $f(-x) = -f(x) = -x^2 + x^5$? Thus, this function is neither an even nor an odd function.

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30. origin

31. $(x, -y)$

32. y-axis

33. y-axis, $(-x, y)$

34. $f(x)$, the y-axis

35. odd, origin

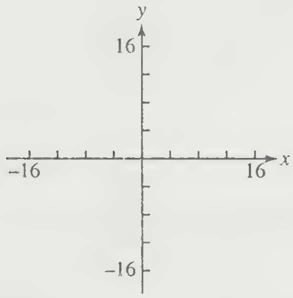
36. $x^2 + x^4$, even

37. $x^2 + x^5$, no, no

OBJECTIVE D: Make a table of values and graph a piecewise defined function.

38. Complete the table of values and graph the function defined by
$$y = \begin{cases} 1-x, & x<0, \\ x^2-1, & x \ge 0. \end{cases}$$

x	−3	−2	−1	0	1	2	3	4
y								



OBJECTIVE E: Define *absolute value* and know its basic properties.

39. The absolute value of a number x is denoted by ____________, and is defined by ____________ if $x \ge 0$ and ____________ if $x < 0$. Thus, $|-3| = -($______$) =$ ____________ and $\left|\frac{2}{3}\right| =$ ____________.

40. Geometrically, $|x|$ represents the distance from __________ to the ______________ on the real line. More generally, $|x-y|$ is the ____________________________________.

41. $\sqrt{x^2} =$ ____________. If you already know $x \ge 0$ you can write $\sqrt{x^2} =$ ____________.

42. The basic absolute value properties are

$|a+b| \le$ ______________

$|ab| =$ ______________

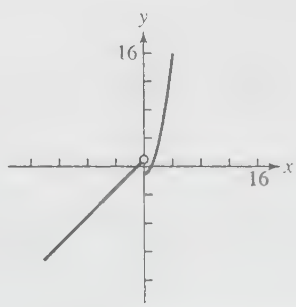
$|-a| =$ ______________

$\left|\frac{a}{b}\right| =$ ______________.

38.

x	−3	−2	−1	0
y	4	3	2	−1

x	1	2	3	4
y	0	3	8	15



39. $|x|, x, -x, -3, 3, \frac{2}{3}$

40. x, origin, distance between x and y

41. $|x|, x$

42. $|a| + |b|, |a|\,|b|, |a|, \frac{|a|}{|b|}$

OBJECTIVE F: Given an equation representing a line, circle, or parabola, write an equation for a shifted graph when the number of units and directions of the shift are specified.

43. The equation $y = f(x) + k$ shifts the graph of f ____________ $|k|$ units if $k < 0$.

44. The equation $y = f(x - h)$ shifts the graph of f ____________ h units if $h > 0$.

45. The equation $y = f(x) + k$ shifts the graph of f ____________ k units if $k > 0$.

46. The equation $y = f(x - h)$ shifts the graph of f ____________ $|h|$ units if $h < 0$.

47. To shift the graph of the line $y = -2x + 1$ horizontally 3 units to the right, we rewrite its equation as

________________________________.

Simplifying algebraically, $y = -2x +$ ______________.

48. To shift the graph of the parabola $y = -x^2$ up 2 units and to the left 1 unit, we rewrite its equation as

________________________________.

Simplifying algebraically $y =$ ________________________.

OBJECTIVE G: Given two functions f and g, write an expression for their composite $f(g(x))$.

49. If $f(x) = 5x + 2$ and $g(x) = x^2$, then a formula for $f(g(x))$ is obtained as follows:

$f(g(x)) = f(x^2) =$ ________________.

The domain of $y = f(g(x))$ is all values of x in the domain of g such that $f(g(x))$ is defined. This is the interval

______________.

50. If $f(x) = \sqrt{x-1}$ and $g(x) = x + 1$, then

$f(g(x)) =$ __________.

The domain of the composite is all values of x in the domain of g such that $f(g(x))$ is defined. This is the interval

__________.

51. Let $f(x) = x^2$ and $g(x) = \sqrt{x-1}$. The $f(g(x)) =$ ______________. The domain of g is the set of all real numbers x satisfying ______________. Thus, the domain of the composite $y = f(g(x))$ is the interval

______________.

OBJECTIVE H: Find two functions f and g that will produce a given composite function h such that $h(x) = f(g(x))$.

52. Consider $h(x) = \sin(x^2 - 1)$. If we let $f(x) = \sin x$ and $g(x) =$ ________________, then

$h(x) =$ ________________ $= f(x^2 - 1) =$ ________________.

53. If $h(x) = \sqrt{x^5 + 2x^3 - 1}$, then for $f(x) = \sqrt{x}$ and $g(x) =$ ________________, it is true that $h(x) = f(g(x))$.

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43. down

44. right

45. up

46. left

47. $y - 1 = -2(x - 3)$, 7

48. $y - 2 = -(x + 1)^2$, $-x^2 - 2x + 1$

49. $5x^2 + 2$, $(-\infty, \infty)$

50. $\sqrt{x}$, $[0, \infty)$

51. $x - 1$, $x \geq 1$, $[1, \infty)$

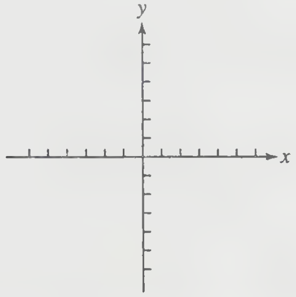
52. $x^2 - 1$, $f(g(x))$, $\sin(x^2 - 1)$

53. $x^5 + 2x^3 - 1$

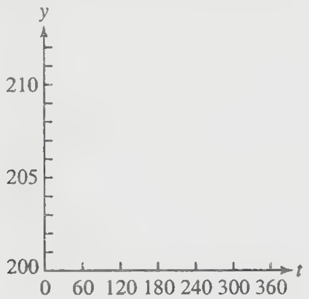
P.3 EXPONENTIAL FUNCTIONS

OBJECTIVE A: Work with functions of the form $f(x) = a^x$, $a > 1$.

54. For $f(x) = 2^x - 3$, $f(-1) =$ ____________, $f(1) =$ ____________, and $f(2) =$ ____________. Graph the function for $-3 \le x \le 3$.



55. \$200 deposited in a bank account grows by 0.015% per day. Graph the account balance over 360 days as $f(t) = 200(1.00015)^t$.



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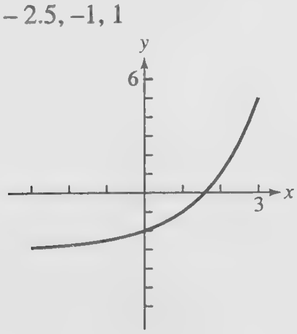
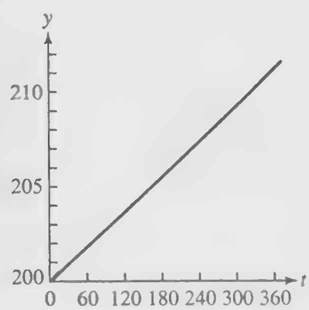
54. −2.5, −1, 1

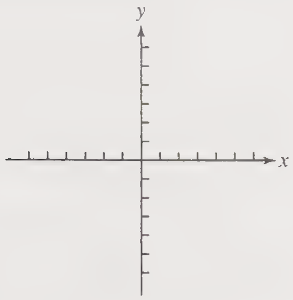


55.

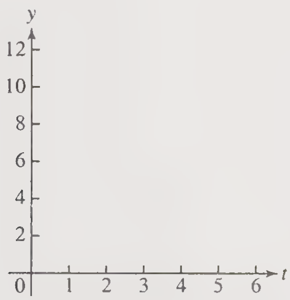


OBJECTIVE B: Work with functions of the form $f(x) = a^x$, $0 < a < 1$.

56. For $f(x) = \left(\frac{1}{3}\right)^x + 1$, $f(-2) =$ __________, $f(-1) =$ __________, and $f(1) =$ __________. Graph the function for $-2 \le x \le 2$.



57. A population of 10 million bacteria is dying off in the presence of a toxin, according to the population function $P(t) = 10\left(\frac{1}{2}\right)^t$, with P in millions and t in hours. Graph $P(t)$ for the first six hours after $t = 0$.



OBJECTIVE C: Know the Rules for Exponents.

58. Simplified, $2^x \cdot a^x =$ __________.

59. Simplified, $(3^b)^2 =$ __________.

60. Simplified, $\frac{9^4}{3^4} =$ __________ $= 3^4 =$ __________.

61. Simplified, $\frac{4^b}{4^y} \cdot 4^x =$ __________.

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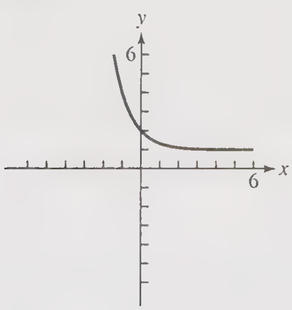
56. $10, 4, \frac{4}{3}$



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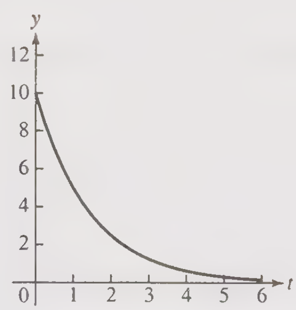


58. $(2a)^x$

59. 3^{2b}

60. $\left(\frac{9}{3}\right)^4$, 81

61. 4^{b+x-y}

OBJECTIVE D: Use the natural exponential function to model exponential growth and decay.

62. A chemical reaction produces a reaction product in an accumulated amount given by the function $A(t) = 2.7e^{1.38t}$, where t is in minutes and A is in milligrams. To find out how much product has accumulated after 5 minutes, substitute 5 for ______ and evaluate the expression ________________ to find that $A(5) =$ ________________ mg.

63. A bank account earns no interest but is subject to a service charge that gradually depletes the account. At $t = 0$, the balance is \$5000, and at later t (in days) the balance is $A(t) = 5000e^{-0.000016t}$. To find the balance after 117 days, substitute ____________ for ____________ and evaluate the expression ______________ to find that $A($______$) =$ ________________.

P.4 INVERSE FUNCTIONS AND LOGARITHMS

OBJECTIVE A: Find the inverse for a specified function.

64. If f and g are inverse functions on suitably restricted domains, then $g(f(x)) =$ ________ and $f(g(y)) =$ ________. That is, the composite of g and f or of f and g is the ______________________ function.

65. Given a function $y = f(x)$, to find a formula for the inverse function f^{-1}, solve the equation $y = f(x)$ for ______ in terms of ________. Interchange the letters x and y. The resulting formula is the inverse ________________.

66. To calculate the inverse of $y = -6x + 2$, interchange the letters x and y obtaining ______________. Solving the resultant equation for y yields ____________________, or $f^{-1}(x) =$ ____________________ is the inverse function of $f(x) = -6x + 2$.

67. If $y = 4 - 7x$, then the inverse function is $f^{-1}(x) =$ ______________.

68. If $y = \sqrt{x+2} - 5$, then the inverse function is found by solving the equation ________________ for y to obtain $y =$ ____________________, where $x \geq$ ________.

69. A common feature of $y = x$, $y = 3 - x$, and $y = \dfrac{1}{x-2} +$ ________ is that in each case the inverse of the function equals ____________________.

OBJECTIVE B: Use the Properties of Logarithms to rewrite expressions containing logarithms.

70. $\log_a bx =$ ______________________ for $b > 0$ and $x > 0$.

71. $\log_{10} \dfrac{100}{y} = \log_{10}$ ______ $- \log_{10}$ ______ $=$ ______________

---

62. t, $2.7e^{1.38 \cdot 5}$, 2679.14

63. 117, t, $e^{-0.000016 \cdot 117}$, 117, \$4990.65

64. x, y, identity

65. x, y, $y = f^{-1}(x)$

66. $x = -6y + 2$, $y = -\frac{1}{6}(x-2)$, $-\frac{1}{6}(x-2)$

67. $\frac{1}{7}(4 - x)$

68. $x = \sqrt{y+2} - 5$, $(x+5)^2 - 2$, 5

69. 2, the function

70. $\log_a b + \log_a x$

71. 100, y, $2 - \log_{10} y$

72. $\log_5 25^x =$ ______________ = ______________

73. $5^{2\log_5 10} = 5^{\log_5 \text{\_\_\_\_}} =$ ______^{2} = __________

OBJECTIVE C: Solve equations with logarithms.

74. To solve $4^{\log_2(4)} - 3^{\log_9(9)} = 5^{\log_5(x) - \log_5(2)}$ for x, first simplify the exponent on the right side to obtain $4^{\log_2(4)} - 3^{\log_9(9)} =$ ______________. This equation simplifies to __________ − __________ = __________, which can be solved to find $x =$ ______________.

75. To find out how long it will take an initial bank deposit of \$1230, compounded continuously at 6.3% annual interest, to grow to \$5000, solve the equation ______________________, where t is in years. After one divides both sides by 1240 and takes the natural logarithm of both sides, the equation becomes ______________. Then $t =$ ______________ ≈ ______________ years.

P.5 TRIGONOMETRIC FUNCTIONS AND THEIR INVERSES

OBJECTIVE A: Convert radian measure to degree measure, and vice versa.

76. The radian measure of 180° is ____________ units. Here the symbol ______________ represents a real number. This real number corresponds to the length that is subtended by an ____________ of a circle of radius 1 with central angle ____________. Therefore, 1° corresponds to ____________ radians, and 1 radian corresponds to ____________ degrees.

77. Converting degree to radian measure, 60° = ____________ radians, − 45° = ______________ radians, and 72° = ______________ radians.

78. Converting from radian to degree measure, $\frac{\pi}{6}$ radians = __________ degrees, $-\frac{3\pi}{2}$ radians = __________ degrees, and 2 radians = ____________ degrees.

Remark. Whenever you encounter sin 2, for instance, you must think the sine of 2 *radians*, not the sine of 2 degrees. The latter is written sin 2°.

OBJECTIVE B: Given an angle in radians, calculate the values of its sine, cosine, tangent, cotangent, secant, and cosecant.

79. $\sin \frac{\pi}{2} =$ ______________.

80. $\tan \frac{\pi}{4} =$ ______________.

81. $\sec \frac{\pi}{3} =$ ______________.

82. $\cos\left(-\frac{\pi}{6}\right) =$ ______________.

---

72. $x \log_5 25, 2x$

73. 10^2, 10, 100

74. $5^{\log_5(x/2)}$, 16, 3, $\frac{x}{2}$, 26

75. $5000 = 1230e^{0.063t}$, $\ln\left(\frac{500}{123}\right) = 0.063t$, $\ln\frac{\left(\frac{500}{123}\right)}{0.063}$, 22.26

76. π, π, arc, 180°, $\frac{\pi}{180}$, $\frac{180}{\pi}$

77. $\frac{\pi}{3}, -\frac{\pi}{4}, \frac{2\pi}{5}$

78. 30, –270, $\left(\frac{360}{\pi}\right)$

79. 1

80. 1

81. 2

82. $\frac{\sqrt{3}}{2}$

83. $\cot\left(-\frac{\pi}{6}\right) =$ ____________.

84. $\csc\left(-\frac{\pi}{3}\right) =$ ____________.

OBJECTIVE C: Know from memory the most important trigonometric formulas.

Problems 85–94 give the most important trigonometric formulas to remember.

85. $\sin(A + B) =$ ____________________.

86. $\cos(A + B) =$ ____________________.

87. $\sin(-x) =$ ________________.

88. $\cos(-x) =$ ________________.

89. $\sin^2\theta + \cos^2\theta =$ ______________.

90. Dividing both sides of the equation in Problem 89 by $\cos^2\theta$ gives $\tan^2\theta + 1 =$ ______________.

91. $\cos 2\theta =$ ________________.

92. $\sin 2\theta =$ ________________.

93. $\cos^2\theta =$ ________________.

94. $\sin^2\theta =$ ________________.

95. You can use the above results to calculate new formulas. For example,

$$\begin{aligned}\sin\left(x - \frac{\pi}{2}\right) &= \sin\left[x + \left(-\frac{\pi}{2}\right)\right] \\ &= \sin x \cos\left(-\frac{\pi}{2}\right) + \underline{\qquad\qquad} \\ &= \sin x \cos\left(\frac{\pi}{2}\right) - \underline{\qquad\qquad} \\ &= (\sin x)\cdot 0 - \underline{\qquad\qquad} \\ &= \underline{\qquad\qquad}.\end{aligned}$$

96. If a, b, and c are the sides of a triangle ABC and if θ is opposite c, then the *law of cosines* is the equation ____________________.

OBJECTIVE D: Find the values of inverse trigonometric functions at selected points without the use of tables or a calculator.

97. $y = \sin^{-1} x$ is equivalent to ____________, where ________ $\le x \le$ ________ and ________ $\le y \le$ ________.

---

83. $-\sqrt{3}$

84. $-\frac{2}{\sqrt{3}}$

85. $\sin A \cos B + \cos A \sin B$

86. $\cos A \cos B - \sin A \sin B$

87. $-\sin x$

88. $\cos x$

89. 1

90. $\sec^2\theta$

91. $\cos^2\theta - \sin^2\theta$

92. $2\sin\theta\cos\theta$

93. $\frac{1+\cos 2\theta}{2}$

94. $\frac{1-\cos 2\theta}{2}$

95. $\cos x \sin\left(-\frac{\pi}{2}\right)$, $\cos x \sin\frac{\pi}{2}$, $(\cos x)\cdot 1$, $-\cos x$

96. $c^2 = a^2 + b^2 - 2ab\cos\theta$

97. $x = \sin y$, -1, 1, $-\frac{\pi}{2}$, $\frac{\pi}{2}$

98. $y = \tan^{-1} x$ is equivalent to __________, where ________ $< x <$ ________ and ________ $< y <$ ________.

99. Let $y = \sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$; then $\sin y =$ ________, so $y =$ ________. That is, $y = \sin^{-1}\left(-\frac{\sqrt{2}}{2}\right) =$ ________.

100. If $\alpha = \tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)$, then $\tan \alpha =$ ________, so $\alpha =$ ________. Hence, $\sin \alpha =$ ________ and $\cos \alpha =$ ________.

OBJECTIVE E: Evaluate expressions involving inverse trigonometric functions and trigonometric functions.

101. To find $\sin\left(\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right)$, let $y = \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$. Then, $\cos y =$ __________ so $y =$ __________. Hence,

$\sin y =$ __________. Alternatively, since $\sin^2 y + \cos^2 y = 1$ holds,

$$\sin\left(\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right) = \sqrt{1 - \cos^2\left(\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right)}$$

$$= \left[1 - \cos^2\left(\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right)\right]^{1/2}$$

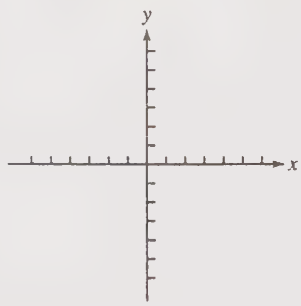
$$= \sqrt{1 - \underline{\qquad\qquad}} = \underline{\qquad\qquad}.$$

P.6 PARAMETRIC EQUATIONS

OBJECTIVE A: Given parametric equations $x = f(t)$ and $y = g(t)$ for the motion of a particle in the xy-plane, eliminate the parameter t to find a Cartesian equation for the particle's path. Graph the Cartesian equation.

102. Consider the curve given by the parametric equations $x = t - 2$, $y = 2t + 3$, $-\infty < t < \infty$. Complete the following table providing some of the points $P(x, y)$ on the curve:

t	−2	−1	0	1	2	3
x	−4					
y	−1					



To elimate the parameter t, note that $t = x + 2$. Substitution for t in the parametric equation for y gives $y =$ __________. This is a cartesian equation for a ____________ with slope $m =$ __________ and y-intercept $b =$ ________.

Sketch the curve in the coordinate system to the right.

---

98. $x = \tan y, -\infty, \infty, -\frac{\pi}{2}, \frac{\pi}{2}$

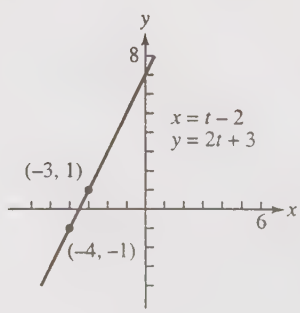
99. $-\frac{\sqrt{2}}{2}, -\frac{\pi}{4}, -\frac{\pi}{4}$

100. $-\frac{\sqrt{3}}{3}, -\frac{\pi}{6}, -\frac{1}{2}, \frac{\sqrt{3}}{2}$

101. $-\frac{\sqrt{3}}{2}, \frac{5\pi}{6}, \frac{1}{2}, \frac{3}{4}, \frac{1}{2}$

102.

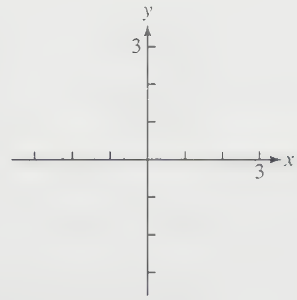
t	−2	−1	0	1	2	3
x	−4	−3	−2	−1	0	1
y	−1	1	3	5	7	9



$2x + 7$, line, 2, 7

103. For the curve given by the parametric equations $x = e^t$ and $y = e^{-t}$, $-\infty < t < \infty$, complete the following table:

t	-2	-1	0	1	2	3
x						
y						



To elimate the parameter t, notice that $xy =$ ________. This equation describes a ______________. Sketch the graph in the coordinate system at the right.

104 For the curve given parametrically in Problem 103, notice that x and y are always positive. Are the parametric equations and the cartesian equation coextensive? ______________, because x and y can both be ______________________ in the cartesian equation $xy = 1$.

OBJECTIVE B: Find parametric equations for a curve described geometrically, or by an equation, in terms of some specified or arbitrary parameter.

105. Find parametric equations for the circle with center $C(-2, 3)$ and radius $r = \sqrt{2}$.
Solution. An equation of the circle is $(x+2)^2 +$ ____________ $=$ __________, or ____________________ $= 1$.
This suggests the substitutions $\frac{x+2}{\sqrt{2}} = \sin\theta$ and $\frac{y-3}{\sqrt{2}} =$ ____________. Hence, parametric equations for the circle are $x =$ _______________ and $y =$ _______________, $0 \le \theta \le 2\pi$.

106. Find parametric equations for the line in the plane through the point (a, b) with slope m, where the parameter t is the change $x - a$.
Solution. For any point $P(x, y)$ on the line, $y - b = m($_________$) =$ ___________. Thus, $x =$ __________ and $y =$ ____________ give parametric equations of the line in terms of the specified parameter t.

P.7 MODELING CHANGE

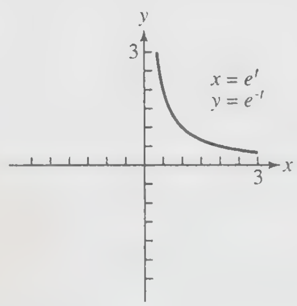
OBJECTIVE A: Use proportionality to model relationships between physical variables.

107. If y is proportional to x, and $y = 8$ when $x = 32$, then the constant of proportionality is ____________, and when $x = 4$, $y =$ ____________.

103.

t	-2	-1	0	1	2	3
x	0.14	0.37	1	2.7	7.4	20
y	7.4	2.7	1	0.37	0.14	0.05

(approximate values)



1, hyperbola

104. No, negative

105. $(y-3)^2$, 2, $\left(\frac{x+2}{\sqrt{2}}\right)^2 + \left(\frac{y-3}{\sqrt{2}}\right)^2$, $\cos\theta$, $\sqrt{2}\sin\theta - 2$, $3 + \sqrt{2}\cos\theta$

106. $x - a$, mt, $a + t$, $b + mt$

107. $\frac{1}{4}$, 1

108. The table below gives the relationship between the weight and the cost of a product.

Weight (lb)	3.0	4.5	7.6	10.1	15.5
Cost (dollars)	10.80	16.20	27.36	36.36	55.80

The relationship can be summarized by the equation cost = _______________.

OBJECTIVE B: Understand the limitations of mathematical models.

109. Making predictions that go beyond the range of the available data is called _______________. While sometimes appropriate, it can lead to erroneous predictions, especially if there is no _______________ for the model.

---

108. 3.6 × weight

109. extrapolation, rationale

PRELIMINARIES CHAPTER SELF-TEST

1. A particle moves in the plane along a straight line from $P(-3, -1)$ to $Q(7, -3)$. Find the net changes $\triangle x$ and $\triangle y$ and distance from P to Q.

2. A particle moves from the point $A(2, -3)$ to the x-axis in such a way that $\triangle y = -6\triangle x$. What are its new coordinates?

3. Determine if the points $A(1, -3)$, $B(-2, 9)$, and $C(5, -19)$ lie exactly along a single straight line.

4. Find the slope of the line through the points $(1, 4)$ and $(-3, 2)$, and write an equation of the line.

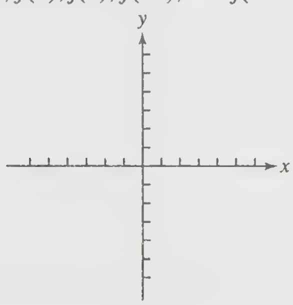
5. Determine the slope, the x-intercept, and the y-intercept for each of the following equations:

(a) $3x + 4y = -1$ **(b)** $x = 2$ **(c)** $y = -1$

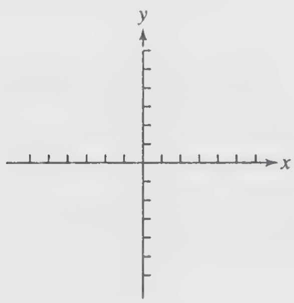
(d) $x^2 = 2y - 1$

6. Find an equation of the line through the point $(5, -7)$ and perpendicular to the line $2y - x = 8$.

7. Let f be defined by the equation $f(x) = x^2 + 3x - 2$. Find the domain and range of f. Also find the values of $f(-2), f(-1), f(0), f(2), f(2b)$, and $f(a + b)$, and sketch the graph of f.



8. Find the domain and range of the function $f(x) = \dfrac{x^2 - x - 6}{x + 2}$ and sketch the graph.



9. Investigate any symmetries of the equation $x^{2/3} + y^{2/3} = 1$.

10. Say whether the functions are even, odd, or neither.

(a) $y = 1 + 2\sin x$ **(b)** $y = |x| \cos 2x$

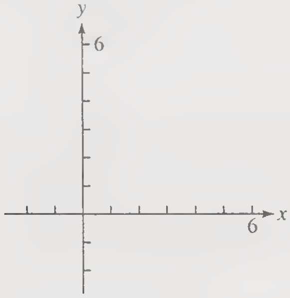
11. The following tell how many units and in what directions the graphs of the given equations are to be shifted. Give an equation for the shifted graph.

(a) $y = x^2$, up 2, left 3 **(b)** $y = \sin x$, down 3, right $\dfrac{\pi}{4}$

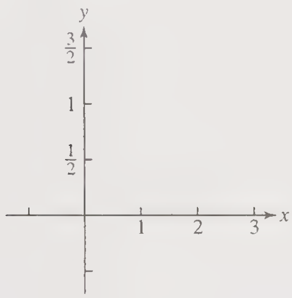
12. Write an equation of the circle with center $(1, -3)$ and radius 4.

13. Find the coordinates of the center and the radius of the circle $x^2 + y^2 + 2x - 4y - 40 = 0$.

14. Graph the parabola and label the vertex, axis, and intercepts: $y = x^2 - 4x + 5$.



15. Graph the function $y = \frac{1}{2}\sin(3x-2) + \frac{1}{2}$.



16. Determine the following values.

(a) $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ **(b)** $\tan^{-1}\frac{\sqrt{3}}{3}$ **(c)** $\tan^{-1}\left(\cos\frac{\pi}{2}\right)$

(d) $\cos^{-1}\left(\sin\frac{\pi}{6}\right)$

17. In the parametrization $x = 3t + 1$, $y = t^2$, $0 \le t \le 6$, eliminate the parameter t and write y as a function of x.

18. Find parametric equations, using θ as the parameter, $0 \le \theta \le 2\pi$, for a circle with radius 5 and center at $(-11, 7)$.

19. Given that $y = kx$, where k is a constant of proportionality, if $k = 2.5$ then the relationship between x and y can be described by saying, in English, that ______________________.

SOLUTIONS TO PRELIMINARIES CHAPTER SELF-TEST

1. $\Delta x = 7 - (-3) = 10$, $\Delta y = -3 - (-1) = -2$, $d = \sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{104} \approx 10.198$

2. The new coordinates can be written as $(x, 0)$ since the point lies on the x-axis. From $\Delta y = -6\Delta x$, we have $0 - (-3) = -6(x - 2)$ or, solving, $x = \frac{3}{2}$. Thus, $\left(\frac{3}{2}, 0\right)$ gives the coordinates of the new position of the particle.

3. The slope of AB is $m_1 = \frac{9-(-3)}{-2-1} = -4$ and the slope of BC is $m_2 = \frac{-19-9}{5-(-2)} = -4$. Since these slopes are equal, the three points do lie along a single straight line.

4. $m = \frac{2-4}{-3-1} = \frac{1}{2}$ is the slope, and $y = 4 + \frac{1}{2}(x-1)$ or $2y - x = 7$ is an equation of the line.

5. **(a)** Solving algebraically for y, $y = -\frac{3}{4}x - \frac{1}{4}$. Thus, the slope is $m = -\frac{3}{4}$, the y-intercept is $b = -\frac{1}{4}$; and when $y = 0$, $x = -\frac{1}{3}$ is the x-intercept.

(b) This is a vertical line so it has no slope and no y-intercept. The x-intercept is 2.

(c) This is a horizontal line. It has slope 0 and y-intercept -1. It has no x-intercept.

(d) Since the variable x is squared, this equation does not represent a straight line. When $x = 0$, $y = \frac{1}{2}$ so the y-intercept is $\frac{1}{2}$. If $y = 0$, $x^2 = -1$ which is impossible, so it has no x-intercept.

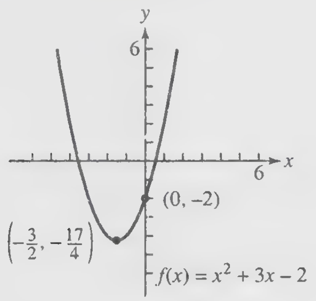
6. The slope of $2y - x = 8$ or $y = \frac{1}{2}x + 4$ is $m = \frac{1}{2}$. Therefore, the slope of the perpendicular line is $m_{\perp} = -2$ and an equation is given by $y + 7 = -2(x - 5)$ or $y = -2x + 3$.

7. The function $f(x) = x^2 + 3x - 2$ is defined for all values of x, so the domain is $-\infty < x < \infty$ (all real numbers). Setting $y = x^2 + 3x - 2$ or $y + 2 = x^2 + 3x$ and completing the square on the righthand side gives

$$y + 2 + \frac{9}{4} = \left(x + \frac{3}{2}\right)^2, \text{ or } y + \frac{17}{4} = \left(x + \frac{3}{2}\right)^2.$$

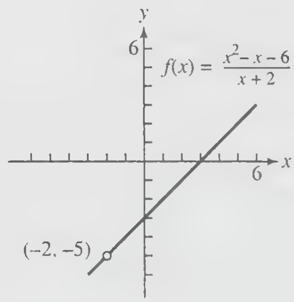
Thus, $y \geq -\frac{17}{4}$, so the range of f is the interval $\left[-\frac{17}{4}, \infty\right)$.

$f(-2) = -4$, $f(-1) = -4$,
$f(0) = -2$, $f(2) = 8$,
$f(2b) = 4b^2 + 6b - 2$,
$f(a + b) = a^2 + 2ab + b^2 + 3(a + b) - 2$
The graph of f is shown below.

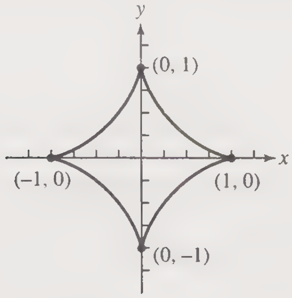


8. $\frac{x^2 - x - 6}{x + 2} = \frac{(x - 3)(x + 2)}{x + 2}$.
Thus, the domain of f is all real numbers except $x = -2$. Also, for $x \neq -2$, $f(x) = x - 3$. This is a straight line with the point $(-2, -5)$ deleted so the range of f is all real numbers except $y = -5$.

The graph of f is shown at the right.



9. Since $x^{2/3}+y^{2/3}=1$ can be written $(x^{1/3})^2+(y^{1/3})^2=1$, the curve is symmetric with respect to both axes and the origin. Also, $1-x^{2/3}$ and $1-y^{2/3}$ must both be nonnegative, so $-1 \le x \le 1$ and $-1 \le y \le 1$ defines the extent of the curve in the x and y directions. The points (–1, 0), (1, 0), (0, –1), and (0, 1) are the intercepts.



10. **(a)** neither **(b)** even

11. **(a)** $y-2=(x+3)^2$ **(b)** $y+3=\sin\left(x-\frac{\pi}{4}\right)$

12. $(x-1)^2+(y+3)^2=16$ or $x^2+y^2-2x+6y-6=0$.

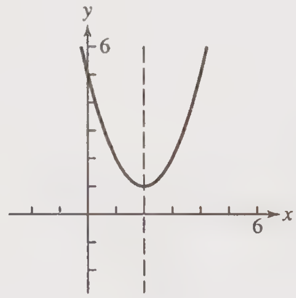
13. Completing the square, $(x+1)^2+(y-2)^2=45=(3\sqrt{5})^2$. The center is (–1, 2) and the radius is $3\sqrt{5}$.

14. $y=x^2-4x+5=(x-2)^2+1$ or $y-1=(x-2)^2$

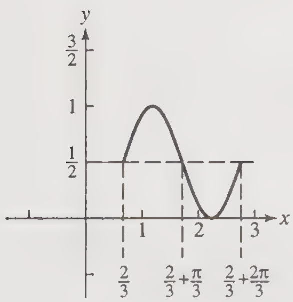
Vertex: (2, 1)

Axis of symmetry: $x=-\dfrac{(-4)}{2(1)}=2$

Intercepts: (0, 5), no x-intercepts



15.



16. **(a)** $-\dfrac{\pi}{3}$ **(b)** $\dfrac{\pi}{6}$ **(c)** 0

(d) $\dfrac{\pi}{3}$

17. $y=\left(\dfrac{x-1}{3}\right)^2,\ 1 \le x \le 19$

18. $x=-11+5\cos\theta,\ y=7+5\sin\theta$

19. y is always 2.5 times as much as x.