# Conjugate functions

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Let us coinsider a vector space $E$ and $E'$ is its algebraic dual. The evaluation map is the bilinear function

$$< E, E' > \to R, < x, x' > = x'(x)$$

and $f : E \to R$. The conjugate $f'$ of $f$ is defined in the following way:

$$f'(x') = \sup\{x'(x) - f(x), x \in E\},$$

either on some subset $B$ of $E$, which is lineally bounded and lineally closed. If $f$ is convex, then $-f$ is concave. Then $f'$ is concave. If $f$ is strictly convex, then $f'$ is strictly concave. In this case, $-f'$ is a strictly convex function. $f$ may takes infinity values, but it is finite valued for some pure subset of $E$ and especially on $B$.