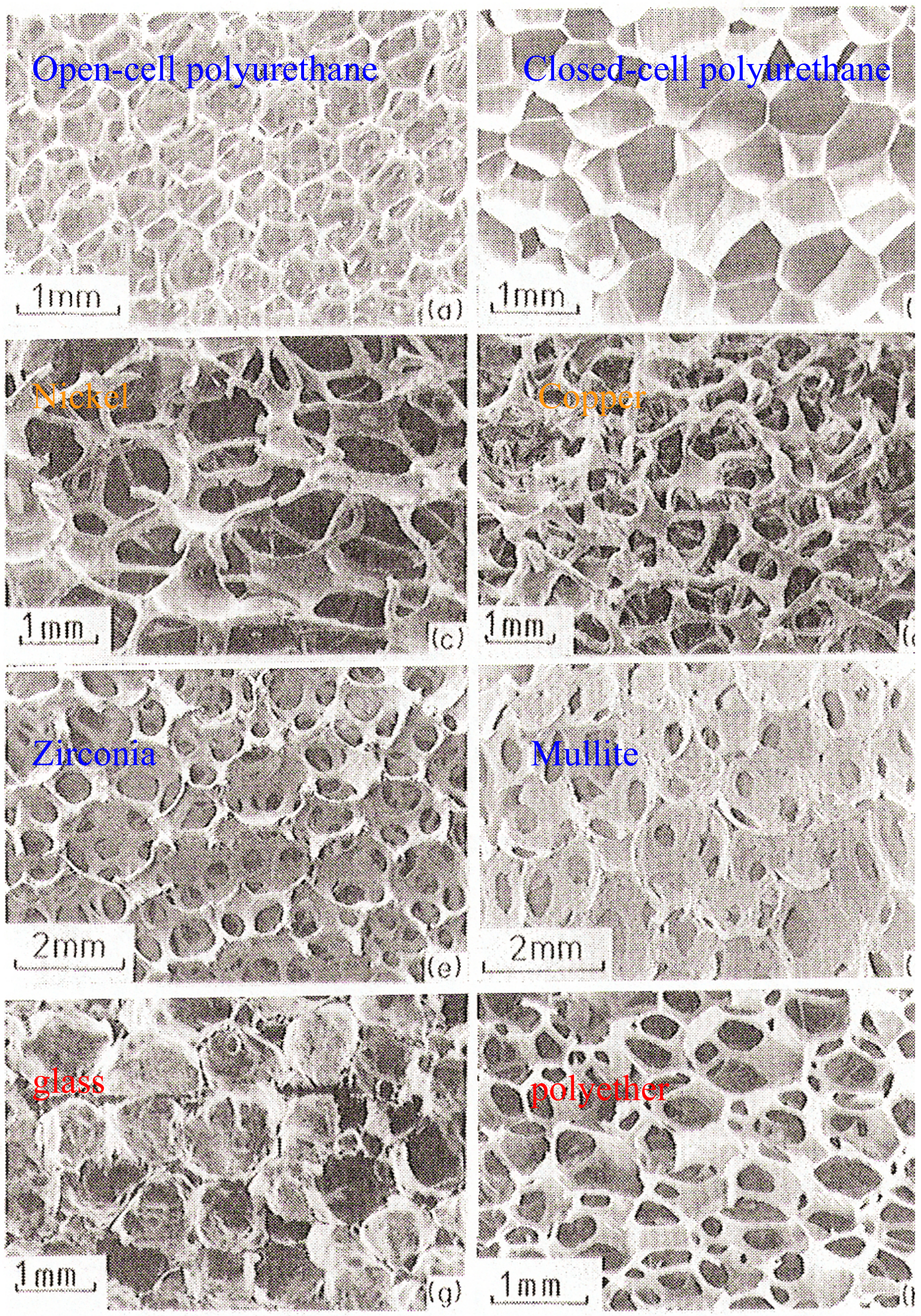
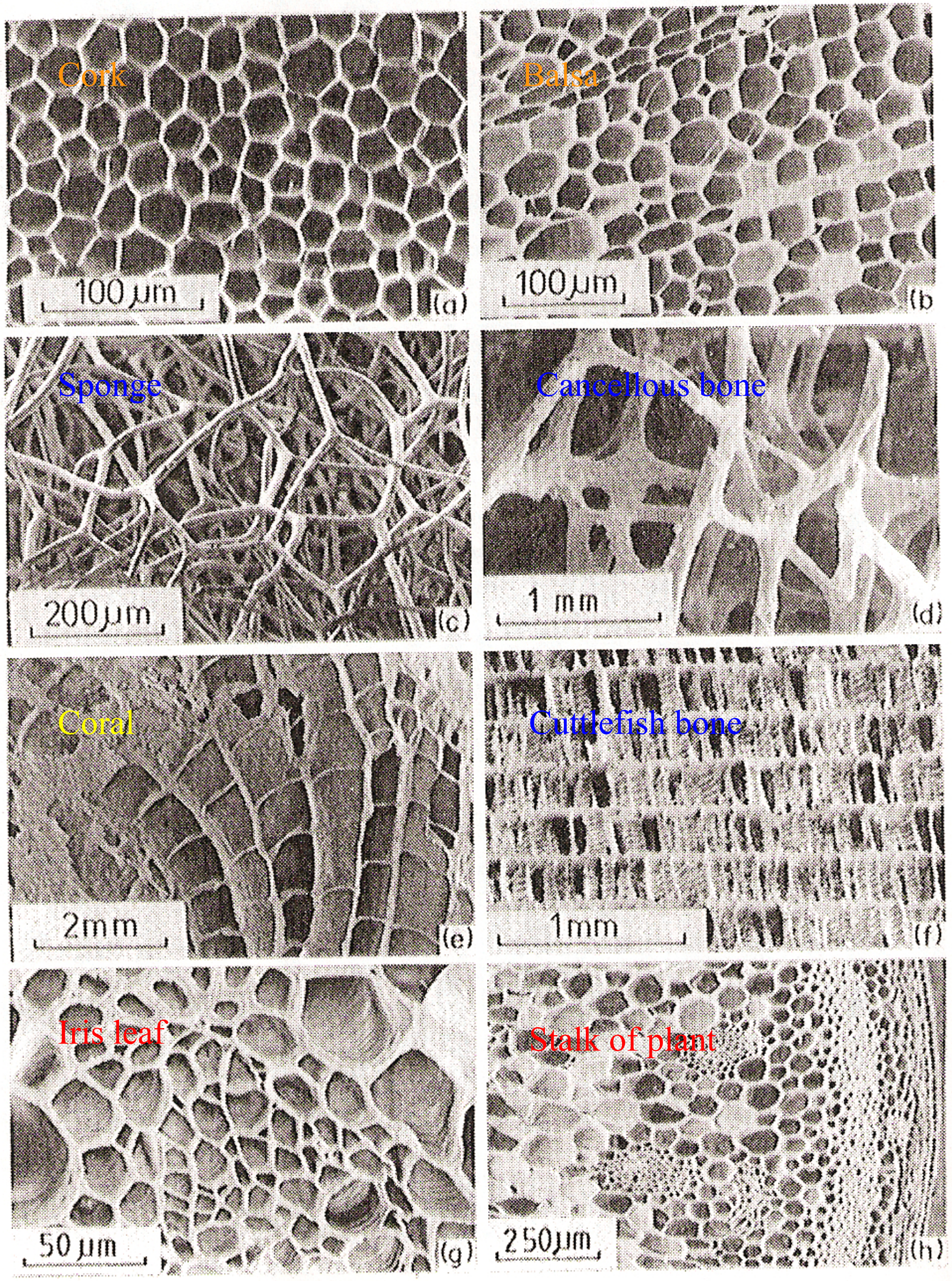
Artificial three dimensional cellular materials:

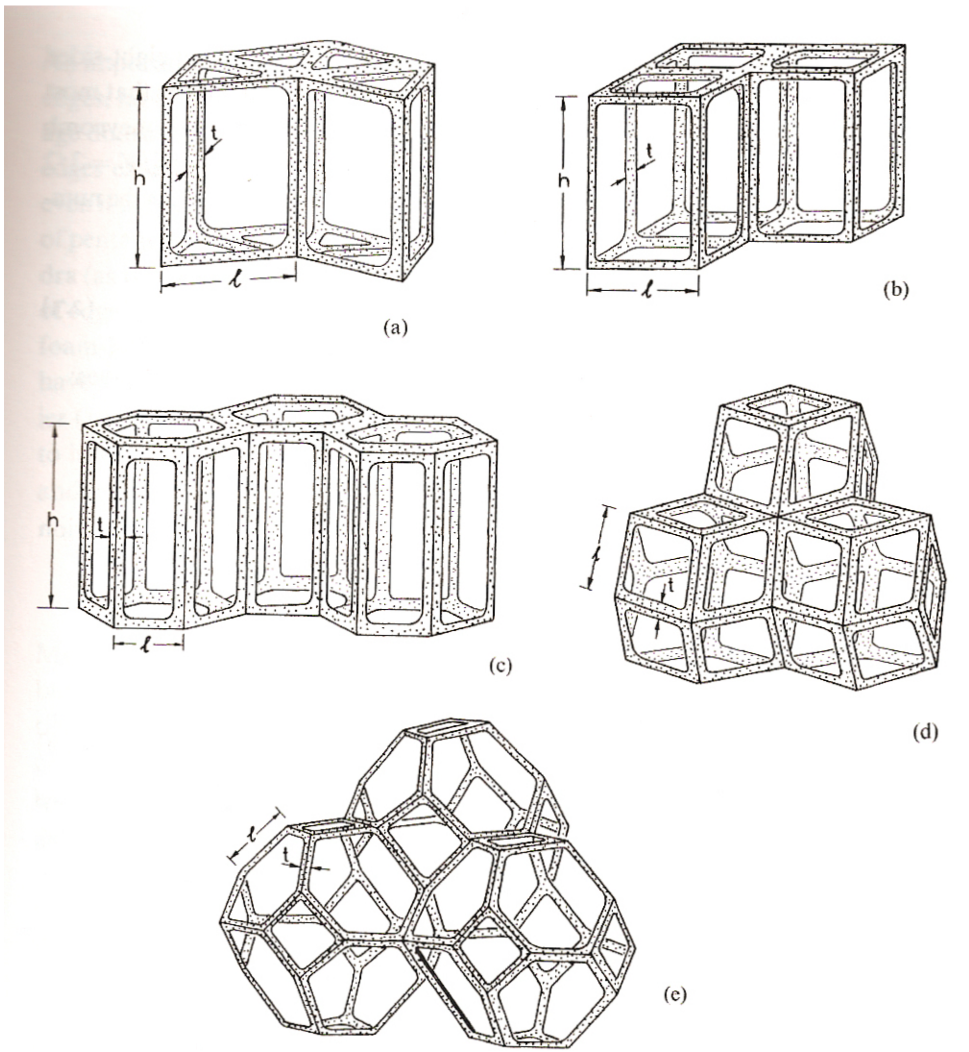


Natural three dimensional cellular materials:



Space filling shapes in two and three dimensions

Only a few shapes are space filling



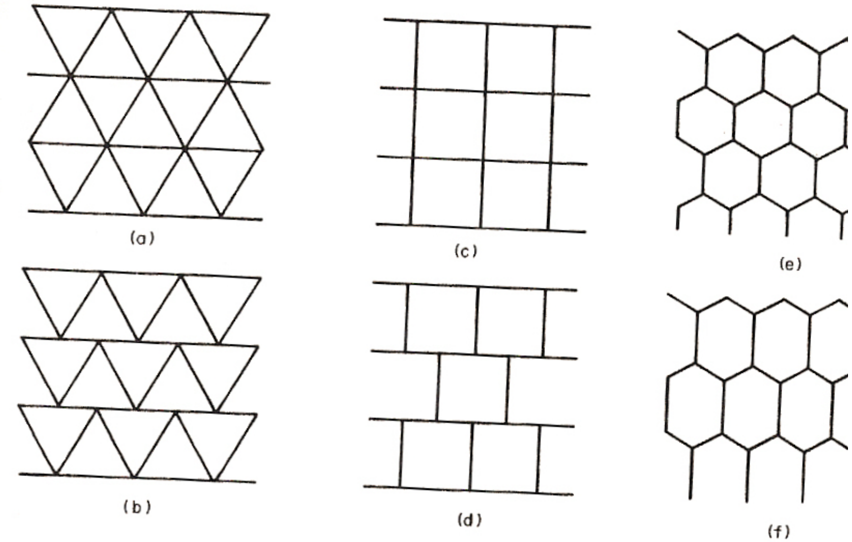


Figure 2.11 Packing of two-dimensional cells to fill a plane: (a, b) Two packi of equilateral triangles with $Z_e = 6$ and $Z_e = 4$, respectively. When $Z_e = 4$, n = topologically. (c, d) Two packings of squares with $Z_e = 4$ and $Z_e = 3$, respectiv When $Z_e = 3$, $n = 6$ topologically. (e) Packing of regular hexagons. (f) Packin of irregular hexagons.

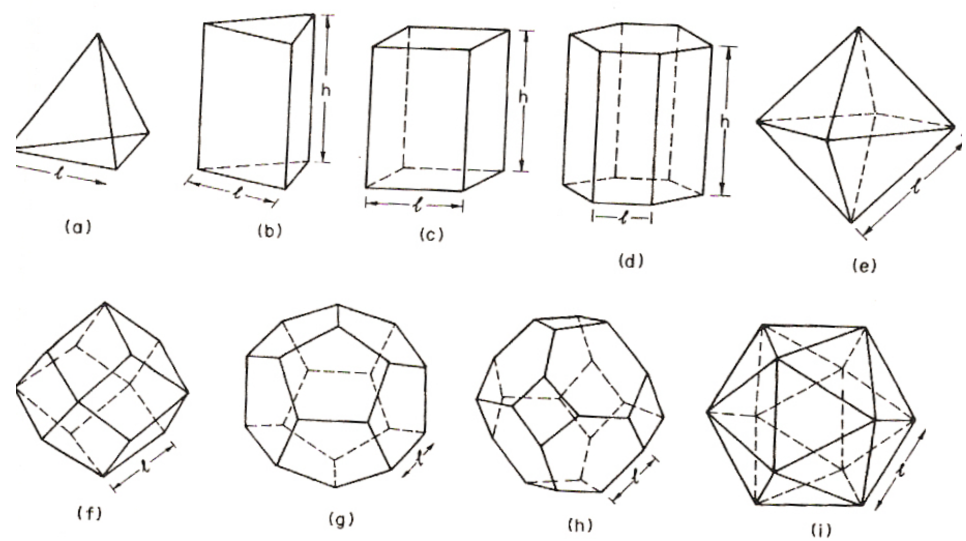


Figure 2.12 Three-dimensional polyhedral cells: (a) tetrahedron, (b) triangular prism, (c) rectangular prism, (d) hexagonal prism, (e) octahedron, (f) rhombic dodecahedron, (g) pentagonal dodecahedron, (h) tetrakaidecahedron, (i) icosahedron.

Relative density

nomenclature

Z_e: number of edges that meet at a vertex

Z_f: number of faces that meet at an edge

n: number of edges per face

$\bar{n}$: average number of edges per face

f: number of faces per cell

$\bar{f}$: average number of faces per cell

Table 2.2 The relative densities of cell aggregates

Honeycombs	
Equilateral triangles ($Z_e = 6, n = 3$ or $Z_e = 4$, or $n = 4$)†	$\frac{\rho^*}{\rho_s} = 2\sqrt{3}\,\frac{t}{l}\left(1 - \frac{\sqrt{3}}{2}\frac{t}{l}\right)$
Squares ($Z_e = 4, n = 4$ or $Z_e = 3, n = 6$)†	$\frac{\rho^*}{\rho_s} = 2\,\frac{t}{l}\left(1 - \frac{1}{2}\frac{t}{l}\right)$
Regular hexagons ($Z_e = 3, n = 6$)	$\frac{\rho^*}{\rho_s} = \frac{2}{\sqrt{3}}\frac{t}{l}\left(1 - \frac{1}{2\sqrt{3}}\frac{t}{l}\right)$
Three dimensions: open cells (aspect ratio $A_r = h/l$)	
Triangular prisms ($Z_e = 8, Z_f = 4.5, \bar{n} = 3.6, \bar{f} = 5$)	$\frac{\rho^*}{\rho_s} = \frac{2}{\sqrt{3}}\frac{t^2}{l^2}\left\{1 + \frac{3}{A_r}\right\}$
Square prisms ($Z_e = 6, Z_f = 4, \bar{n} = 4, \bar{f} = 6$)	$\frac{\rho^*}{\rho_s} = \frac{t^2}{l^2}\left\{1 + \frac{2}{A_r}\right\}$
Hexagonal prisms ($Z_e = 5, Z_f = 3.6, \bar{n} = 4.5, \bar{f} = 8$)	$\frac{\rho^*}{\rho_s} = \frac{4}{3\sqrt{3}}\frac{t^2}{l^2}\left\{1 + \frac{3}{2A_r}\right\}$
Rhombic dodecahedra ($Z_e = 5.33, Z_f = 3, \bar{n} = 4, \bar{f} = 12$)	$\frac{\rho^*}{\rho_s} = 2.87\frac{t^2}{l^2}$
Tetrakaidecahedra ($Z_e = 4, Z_f = 3, \bar{n} = 5.14, \bar{f} = 14$)	$\frac{\rho^*}{\rho_s} = 1.06\frac{t^2}{l^2}$
Three dimensions: closed cells (aspect ratio $A_r = h/l$)	
Triangular prisms ($Z_e = 8, Z_f = 4.5, \bar{n} = 3.6, \bar{f} = 5$)	$\frac{\rho^*}{\rho_s} = 2\sqrt{3}\,\frac{t}{l}\left\{1 + \frac{1}{2\sqrt{3}A_r}\right\}$
Square prisms ($Z_e = 6, Z_f = 4, \bar{n} = 4, \bar{f} = 6$)	$\frac{\rho^*}{\rho_s} = 2\,\frac{t}{l}\left\{1 + \frac{1}{2A_r}\right\}$
Hexagonal prisms ($Z_e = 5, Z_f = 3.6, \bar{n} = 4.5, \bar{f} = 8$)	$\frac{\rho^*}{\rho_s} = \frac{2}{\sqrt{3}}\frac{t}{l}\left\{1 + \frac{\sqrt{3}}{2A_r}\right\}$
Rhombic dodecahedra ($Z_e = 5.33, Z_f = 3, \bar{n} = 4, \bar{f} = 12$)	$\frac{\rho^*}{\rho_s} = 1.90\,\frac{t}{l}$
Tetrakaidecahedra ($Z_e = 4, Z_f = 3, \bar{n} = 5.14, \bar{f} = 14$)	$\frac{\rho^*}{\rho_s} = 1.18\,\frac{t}{l}$

† See Fig. 2.11.

Geometric features of isolated cells

Table 2.1 Geometric properties of isolated cells

Cell shape	Number of faces, f (a)	Number of edges, n (b)	Number of vertices, v (c)	Cell volume (d) (e)	Surface area (a) (d) (e)	Edge length (b) (e)	Comments (f)
Tetrahedron	4	6	4	$0.118l^3$	$\sqrt{3}l^2$	$6l$	Regular
Triangular prism	5	9	6	$\frac{\sqrt{3}}{4} l^3 A_r$	$\frac{\sqrt{3}}{2} l^2(1+2\sqrt{3}A_r)$	$6l(1+A_r/2)$	Packs to fill space
Square prism	6	12	8	$l^3 A_r$	$2l^2(1+2A_r)$	$8l(1+A_r/2)$	Packs to fill space (cube is regular)
Hexagonal prism	8	18	12	$\frac{3\sqrt{3}}{2} l^3 A_r$	$3\sqrt{3}l^2(1+2A_r/\sqrt{3})$	$12l(1+A_r/2)$	Packs to fill space
Octahedron	8	12	6	$0.471l^3$	$3.46l^2$	$12l$	Regular
Rhombic Dodecahedron	12	24	14	$2.79l^3$	$10.58l^2$	$24l$	Packs to fill space
Pentagonal Dodecahedron	12	30	20	$7.663l^3$	$20.646l^2$	$30l$	Regular
Tetrakaidecahedron	14	36	24	$11.31l^3$	$26.80l^2$	$36l$	Packs to fill space
Icosahedron	20	30	12	$2.182l^3$	$8.660l^2$	$30l$	Regular